

QUIZ 19

key

For all questions, assume that $A = \begin{bmatrix} 2 & 5 \\ 4 & 3 \end{bmatrix}$

1. (2 points) Are $x = \begin{bmatrix} 5 \\ -4 \end{bmatrix}$ and $y = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ eigenvectors of the matrix A?

$$Ax = \begin{bmatrix} 2 & 5 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 5 \\ -4 \end{bmatrix} = \begin{bmatrix} 10 - 20 \\ 20 - 12 \end{bmatrix} = \begin{bmatrix} -10 \\ 8 \end{bmatrix} = -2 \begin{bmatrix} 5 \\ -4 \end{bmatrix} \checkmark$$

$$Ay = \begin{bmatrix} 2 & 5 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 + 5 \\ 8 + 3 \end{bmatrix} = \begin{bmatrix} 9 \\ 11 \end{bmatrix} \neq \lambda \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

2. (3 points) Compute the characteristic equation for A.

$$\begin{aligned} \det(A - \lambda I) &= \begin{vmatrix} 2-\lambda & 5 \\ 4 & 3-\lambda \end{vmatrix} = (2-\lambda)(3-\lambda) - 20 \\ &= 6 - 5\lambda + \lambda^2 - 20 \end{aligned}$$

$$\Rightarrow \lambda^2 - 5\lambda - 14 = 0$$

$$(\lambda - 7)(\lambda + 2) = 0$$

3. (2 points) What are the eigenvalues of A?

$$\lambda = 7$$

$$\lambda = -2$$

4. (3 points) Find one eigenvector associated to each distinct eigenvalue.

$$\underline{\underline{\lambda = -2}} \Rightarrow x = \begin{bmatrix} 5 \\ -4 \end{bmatrix}$$

$$\underline{\underline{\lambda = 7}} \Rightarrow Ax = 7x, \quad x = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 5 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 7a \\ 7b \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2a + 5b \\ 4a + 3b \end{bmatrix} = \begin{bmatrix} 7a \\ 7b \end{bmatrix}$$

$$\Rightarrow \begin{aligned} 5b &= 5a \Rightarrow b = a \\ 4a &= 4b \Rightarrow \uparrow \end{aligned}$$

pick $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(in general: $\bar{x} = \begin{bmatrix} a \\ a \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} a, a \in \mathbb{R}$)