

QUIZ 20

Diagonalize the following matrix if possible:

$$A = \begin{bmatrix} 5 & -1 & 0 & -5 \\ 0 & 1 & 6 & -3 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

eigenvalues are
 $\lambda = 5, 1, 4$

$\lambda = 5$

$$A\bar{x} = 5\bar{x}$$

$$\begin{bmatrix} 5 & -1 & 0 & -5 \\ 0 & 1 & 6 & -3 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 5a \\ 5b \\ 5c \\ 5d \end{bmatrix}$$

$$\begin{cases} 5a - b - 5d = 5a \\ b + 6c - 3d = 5b \\ 4c = 5c \\ 4d = 5d \end{cases} \Rightarrow$$

$$b = 5d = 0$$

a can be anything

$$c = 0$$

$$d = 0$$

$$\Rightarrow v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\lambda = 1$

$$5a - b - 5d = a$$

$$b + 6c - 3d = b$$

$$4c = c$$

$$4d = d$$

\Rightarrow

$$b = 4a$$

$$c = d = 0$$

$$\Rightarrow \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} a \\ 4a \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 0 \\ 0 \end{bmatrix} a$$

\uparrow
 v_2

$\lambda = 4$

$$5a - b - 5d = 4a$$

$$b + 6c - 3d = 4b$$

$$4c = 4c$$

$$4d = 4d$$

\Rightarrow

$$a = b + 5d$$

$$\begin{cases} 3b = 6c - 3d \\ b = 2c - d \end{cases}$$

$$\rightarrow a = 2c + 4d$$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 2c + 4d \\ 2c - d \\ c \\ d \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \end{bmatrix} c + \begin{bmatrix} 4 \\ -1 \\ 0 \\ 1 \end{bmatrix} d$$

\uparrow v_3 \uparrow v_4

$$P = \begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & 4 & 2 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

where $A = PDP^{-1}$