Quiz 5

Key

1. (3 points) Find the image of $(3,-5,1)$ under the transformation $T\colon \mathbb{R}^3 \to \mathbb{R}^2$ where T is defined by $T(x_1,x_2,x_3) = (4x_1 - 7x_2, x_3 - 3)$

way 1:

$$T(3,-5,1) = (4\cdot 3 - 7\cdot -5,\ 1-3)$$
$$= (12+35, -2)$$
$$= (47, -2)$$

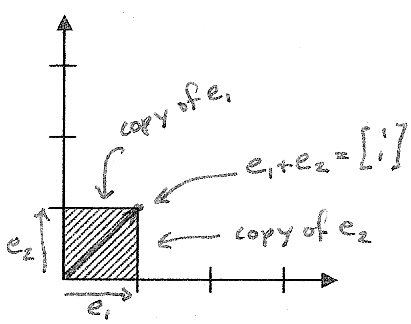
way 2:

$$T\left(\begin{bmatrix}3\\-5\\1\end{bmatrix}\right) = \begin{bmatrix}4\cdot 3 - 7\cdot -5\\ 1-3\end{bmatrix} = \begin{bmatrix}47\\-2\end{bmatrix}$$

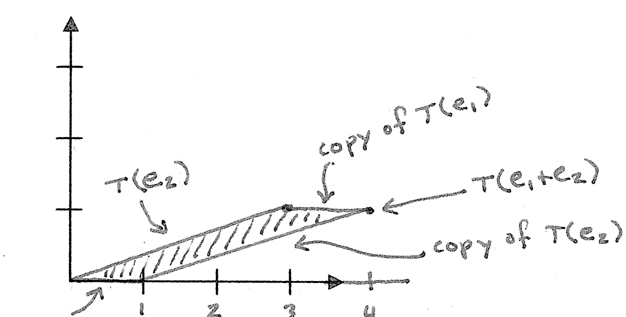
2. (4 points) Find the standard matrix of the linear transformation $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ given that $T(e_1) = (-1,4,-7)$, $T(e_2) = (0,0,3)$, and $T(e_3) = (5,2,3)$. Be careful of rows/columns.

$$A = \begin{bmatrix}T(e_1) & T(e_2) & T(e_3)\end{bmatrix} = \begin{bmatrix}-1 & 0 & 5\\ 4 & 0 & 2\\ -7 & 3 & 3\end{bmatrix}$$

3. (3 points) Find the image of the unit square after the following horizontal shear transformation has been applied to it: $Tx = Ax$, where $A = \begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}$ and $T\colon \mathbb{R}^2 \to \mathbb{R}^2$. Hint: the unit square is formed by using the standard vectors $e_1 = \begin{bmatrix}1\\0\end{bmatrix}$ and $e_2 = \begin{bmatrix}0\\1\end{bmatrix}$.



Unit Square



Shear Transformation

way 1:

transform the sides of the unit square:

$$T(e_1) = \begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}$$

$$T(e_2) = \begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}3\\1\end{bmatrix}$$

way 2:

find where the "points" of the square will end up
ie: $(0,0)$, $(1,0)$, $(0,1)$, & $(1,1)$

$$T\left(\begin{bmatrix}0\\0\end{bmatrix}\right) = \begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$$T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = \begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}4\\1\end{bmatrix}$$

$T(e_1+e_2)$

$$T(e_1) = e_1$$
$$T(e_2) = \begin{bmatrix}3\\1\end{bmatrix}$$