Key

Quiz 7

Determine whether the following matrix is invertible or not. If so, find the inverse. If not, give a reason why no inverse exists.

$$A = \begin{bmatrix} 1 & -7 & 3 \\ 4 & 0 & -1 \\ -2 & 5 & -3 \end{bmatrix}$$

$$[A|I] = \left[\begin{array}{ccc|ccc} 1 & -7 & 3 & 1 & 0 & 0 \\ 4 & 0 & -1 & 0 & 1 & 0 \\ -2 & 5 & -3 & 0 & 0 & 1 \end{array}\right] \begin{matrix} \\ R_2 - 4R_1 \\ R_3 + 2R_1 \end{matrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & -7 & 3 & 1 & 0 & 0 \\ 0 & 28 & -13 & -4 & 1 & 0 \\ 0 & -9 & 3 & 2 & 0 & 1 \end{array}\right] \quad -\tfrac{1}{9}R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & -7 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1/3 & -2/9 & 0 & -1/9 \\ 0 & 28 & -13 & -4 & 1 & 0 \end{array}\right] \begin{matrix} R_1 + 7R_2 \\ \\ R_3 - 28R_2 \end{matrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2/3 & -5/9 & 0 & -7/9 \\ 0 & 1 & -1/3 & -2/9 & 0 & -1/9 \\ 0 & 0 & -11/3 & 20/9 & 1 & 28/9 \end{array}\right] \quad -\tfrac{3}{11}R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2/3 & -5/9 & 0 & -7/9 \\ 0 & 1 & -1/3 & -2/9 & 0 & -1/9 \\ 0 & 0 & 1 & -\frac{20}{33} & -\frac{3}{11} & -\frac{28}{33} \end{array}\right] \begin{matrix} R_1 - \frac{2}{3}R_3 \\ R_2 + \frac{1}{3}R_3 \\ \\ \end{matrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -5/33 & 2/11 & -7/33 \\ 0 & 1 & 0 & -14/33 & -1/11 & -13/33 \\ 0 & 0 & 1 & -20/33 & -3/11 & -28/33 \end{array}\right] = [I|A^{-1}]$$

$\therefore$ $A^{-1} = \begin{bmatrix} -5/33 & 2/11 & -7/33 \\ -14/33 & -1/11 & -13/33 \\ -20/33 & -3/11 & -28/33 \end{bmatrix}$