

Quiz 8

1. (4 points) Determine whether the following matrices can be multiplied and if they can be multiplied as block matrices. If they can be multiplied as block matrices, compute $A \cdot B$ using their submatrices. If not, give a reason why the multiplication can't be performed.

$$A = \begin{bmatrix} 1 & 7 & 3 & 5 & -8 & 2 \\ 4 & 0 & -1 & 7 & 9 & -4 \\ -2 & 5 & -3 & 1 & -3 & 0 \\ 6 & 2 & -2 & 1 & 0 & 7 \\ 7 & 1 & -9 & -8 & 3 & 14 \end{bmatrix}$$

size of $A = 5 \times 6$

$$B = \begin{bmatrix} 1 & 2 & -8 \\ 5 & 4 & -1 \\ 7 & 5 & -2 \\ 0 & 1 & -4 \\ 5 & 1 & 7 \\ 9 & 8 & 10 \end{bmatrix}$$

size of $B = (6 \times 3)$

yes, AB exists

but we can't find AB with these partitions because $A_{11}B_1$ doesn't exist

2. (review topic) Suppose you have the vectors: $v_1, v_2, v_3,$ & v_4 , and you know they are linearly independent. Determine whether the following vectors are linearly independent or linearly dependent?

- a. (3 points) $v_1 - v_2, 3(v_2 - v_3), 2(v_3 - v_4), v_4$

$$c_1(v_1 - v_2) + 3c_2(v_2 - v_3) + 2c_3(v_3 - v_4) + c_4v_4 = 0$$

regroup:

$$c_1v_1 + (-c_1 + 3c_2)v_2 + (-3c_2 + 2c_3)v_3 + (-2c_3 + c_4)v_4 = 0$$

since v_i 's are independent:

$$\begin{aligned} c_1 &= 0 \\ (-c_1 + 3c_2) &= 0 \Rightarrow c_2 = 0 \\ (-3c_2 + 2c_3) &= 0 \Rightarrow c_3 = 0 \\ (-2c_3 + c_4) &= 0 \Rightarrow c_4 = 0 \end{aligned}$$

\therefore these new vectors are also independent

- b. (3 points) $2v_1 - v_2, 2v_2 - 3v_3, v_1, 3v_3 - 4v_1$

$$c_1v_1 + c_2(2v_1 - v_2) + c_3(2v_2 - 3v_3) + c_4(3v_3 - 4v_1) = 0$$

$$(c_1 + 2c_2 - 4c_4)v_1 + (-c_2 + 2c_3)v_2 + (-3c_3 + 3c_4)v_3 = 0$$

$$\Rightarrow \begin{cases} c_1 + 2c_2 - 4c_4 = 0 \\ -c_2 + 2c_3 = 0 \\ -3c_3 + 3c_4 = 0 \end{cases} \Rightarrow \begin{cases} c_1 = -2c_2 + 4c_4 = 0 \\ c_2 = 2c_3 = 2c_4 \\ c_3 = c_4 \end{cases}$$

let $c_4 = 1$

$$\Rightarrow c_1 = 0, c_2 = 2, c_3 = 1$$

$$0 \cdot v_1 + 2(2v_1 - v_2) + 1 \cdot (2v_2 - 3v_3) + 1 \cdot (3v_3 - 4v_1) = 0$$

\therefore these vectors are linearly dependent