**Enter your name(s) here:**

Computer Assignment: Introduction to Mathematica

Purpose: To provide a brief introduction to the *Mathematica* computing environment and to use *Mathematica* to solve systems of linear equations.

Prerequisite Reading: Section 1.1

***Mathematica* Functions:** Clear, MatrixForm, Needs, ReplaceRow, Scale, Swap, TableForm.

Palette used in this assignment: To use the following palette, click on the right cell bracket containing the palette (or click anywhere inside the palette cell) and then choose **General Palette from Selection** from the **File** menu to create a separate window containing the palette. Drag the palette window to the far right side of the screen and resize the notebook window if necessary so that the notebook and palette do not overlap.
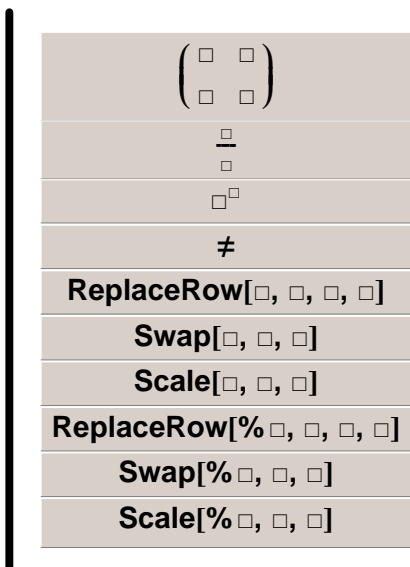
Mathematica Arithmetic

Mathematica can be thought of as a sophisticated calculator, able to perform exact as well as approximate arithmetic computations. As you read through this file, you will be asked to execute the contents of input cells (shaded cells like the one immediately below this paragraph). As a first example, you will add the fractions appearing in the following input cell. Using your mouse, click anywhere inside the following shaded cell and then press **Shift+Enter** to perform the calculation (simply pressing **Enter** may be sufficient to perform the calculation on some machines). **A window will appear asking if you would like to evaluate initialization cells contained in this notebook - click on Yes. (Initialization cells will be discussed at the end of this assignment.)**

$$\frac{552291}{67158} + \frac{225555}{4512874}$$

You may notice that *Mathematica* took a relatively long time to perform this simple calculation. This is due to the fact that on many computer platforms, the bulk of the *Mathematica* program is not loaded into memory until the first *Mathematica* operation is executed.

Also note that the output to our first calculation is an exact answer in rational form (i.e., the output is expressed as the fraction of two integers).

Exercise 1

Compute $\frac{1}{31} + \frac{5}{72}$ with *Mathematica*. To do this, in the open space immediately below this cell and before the following cell entitled *Arithmetic Operations,* click the mouse button and a horizontal line will appear marking the location of the new cell you are about to create. Next, on the palette, click on the button containing $\frac{\Box}{\Box}$. Enter 1, then press **Tab** (or click on the denominator with your mouse) and then enter 31. (Another method is to type in 1/31 from the keyboard.) Next, press the right arrow button ($\rightarrow$) on the keyboard to move the cursor to the right of the fraction. Press the plus (**+**) and then enter the second fraction. Finally, press **Shift+Enter** to execute the command.

Arithmetic Operations

The following table summarizes the *Mathematica* symbols used for addition, subtraction, multiplication, division and powers.

Operation	*Mathematica* Operation
addition	+
subtraction	−
multiplication	∗ or at least one space
division	/ or enter $\frac{\Box}{\Box}$ from the palette
power	^ or enter $\Box^{\Box}$ from the palette
grouping terms	( )

Execute the following command to compute 2 to a power of 500. Notice that the answer is so long that it cannot fit on a single line!

```
2^500
```

A space can be used for multiplication. Therefore, to compute 56987 times 23186, at least one space must be inserted between the two numbers. Execute the following computation.

```
56987 23186
```

Parentheses are used to group terms. *Never use square brackets [ ] or curly brackets { } for grouping in arithmetic or algebraic computations. Always use parenthesis ( ).* For example, execute the following arithmetic operation.

```
5 (8 - 6)  + 2^5
```

Entering a Matrix

Suppose, for example, you wish to enter the matrix $\begin{pmatrix} 1 & 3 & 5 & 9 \\ 2 & 0 & 3 & 7 \\ -5 & 4 & 3 & 2 \end{pmatrix}$ and assign it to the letter A. One way to create this assignment statement is to enter A={{1,3,5,9},{2,0,3,7},{-5,4,3,2}} where each set of inner brackets represents a row of A. Execute the following command to assign A to the given matrix.

```
A = {{1, 3, 5, 9}, {2, 0, 3, 7}, {-5, 4, 3, 2}}
```

To view A in matrix form, execute the following command:

```
MatrixForm[A]
```

Exercise 2

Use brackets { {...}, {...},..., {...} } to create the matrix B = $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 1 & 2 & 13 & 0 \end{pmatrix}$.

A More Natural Method for Creating a Matrix

A more natural way to enter a matrix is to use the palette. Use the mouse and click on the palette button containing $\begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix}$. To insert an additional column, press the comma key while holding the **Ctrl** key. Repeat this action to insert additional columns. To insert rows, press the enter key while holding down the **Ctrl** key (i.e. execute **Ctrl+Enter**). The matrix contained in the following input cell was constructed using this method. Execute the following assignment statement.

```
A = ( 1   3  5  9
      2   0  3  7
     -5   4  3  2 )
```

Once again, the output is not in matrix form. Executing the following command will instruct *Mathematica* to *always* display a matrix in matrix form (you need not understand how this command works, but suffice it to say that before an output statement is displayed, *Mathematica* will check to see if the output is a matrix and if it is, *Mathematica* will display the output in the form of a matrix).

```
$Post := If[MatrixQ[#], MatrixForm[#], #] &
```

Exercise 3

Assign F to equal $\begin{pmatrix} -2.5 & 0 & 2 & 11 \\ 1 & 2 & 3 & 0 \\ -9 & 5 & 7 & -3 \end{pmatrix}$ using the palette. Notice that output will automatically be displayed in matrix form.

Assignment Names

Execute the following assignment statement:

$$D = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & 0 \\ 1 & 0 & 5 \end{pmatrix}$$

When the assignment statement is executed, an error message is displayed indicating that D is a protected symbol. This is due to the fact that **D** is actually a *Mathematica* command used to find the derivative of a function. (Other *Mathematica* commands that you have already seen include **Matrix-Form** and **TableForm**.) *All Mathematica commands are protected symbols and all protected symbols begin with a capital letter.*

So if you create an assignment statement and you obtain a protected symbol error, then you will need to choose a new name in your assignment statement. If you wish to use another capital letter to represent your matrix, avoid using **C**, **D**, **E**, **I**, **N**, and **O** since these are all protected *Mathematica* symbols.

With *Mathematica*, you can use a string of letters for assignment names as long as you begin with a letter of the alphabet and no spaces occur in your assignment statement. An easy way to avoid protected symbol errors is to use an assignment name which begins with a lower case letter. By following this rule of thumb, you will never obtain protected symbol errors since all protected symbols begin with a capital letter. So instead of instead of using the letter **D**, you can use something like **matD**. Execute the following command.

$$matD = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & 0 \\ 1 & 0 & 5 \end{pmatrix}$$

Notice that the matrix is displayed again in an output statement. Placing a semicolon at the end of an assignment statement suppresses the display of the output. Execute the following command.

$$matD = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix};$$

A semicolon can also be used to enter more than one command or assignment statement on a single line. For example, execute the contents of the following cell. The semicolon separates two assignment statements in this case. The semicolon also prevents the output from being displayed.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & -7 \end{pmatrix}; \ B = \begin{pmatrix} -1 & 3 & -2 \\ 2 & 5 & 10 \end{pmatrix};$$

Solving Systems of Linear Equations

Suppose you want to use elementary row operations to solve a system of linear equations:

$$\begin{aligned} 2\,x_1 &+& 4\,x_2 &=& 10 \\ 6\,x_1 &+& 4\,x_2 &=& 14 \end{aligned}$$

To begin, execute the following command to set up the corresponding augmented matrix:

$$M = \begin{pmatrix} 2 & 4 & 10 \\ 6 & 4 & 14 \end{pmatrix}$$

Next you will use the following three *Mathematica* commands to solve the system:

ReplaceRow[M, r, m, s]	Replaces row r of M by row r plus m times row s.
Scale[M, r, c]	Multiplies row r of M by a scalar c.
Swap[M, r, s]	Interchanges rows r and s of M.

The first step in the row reduction of *M* is to replace row 2 by its sum with -3 times row 1. Execute the following command.

```
ReplaceRow[M, 2, -3, 1]
```

Notice that a matrix is now displayed immediately after a statement of the form

Out[n]//MatrixForm=

Determine the specific value of **n** (called the output number) given in this statement. The matrix appearing in this output statement is automatically assigned the name **Out[n]** (**%n** is shorthand for **Out[n]**). For example, if the output number is 19, then the name of the matrix is **%19**.

The next step is to scale row 2 of the matrix just obtained in the last output statement. To do this, place the output number corresponding to the last output statement inside the box (□) in the following scale statement and then execute the command.

```
Scale[% □, 2, -1 / 8]
```

You should obtain an output of $\begin{pmatrix} 2 & 4 & 10 \\ 0 & 1 & 2 \end{pmatrix}$. If not, check to make sure you used the correct output number in the preceding statement. (If not, then reexecute the preceding command after replacing the incorrect output number. To reexecute a command, make the necessary changes to the input cell and then press **Shift+Enter**.) Now place the output number corresponding to this matrix in the box immediately below and then execute the command.

```
ReplaceRow[% □, 1, -4, 2]
```

You should obtain an output of $\begin{pmatrix} 2 & 0 & 2 \\ 0 & 1 & 2 \end{pmatrix}$. If not, check to make sure you used the correct output number in the preceding statement. Now place the output number corresponding to the last output in the following box and then execute the given command.

```
Scale[% □, 1, 1 / 2]
```

The output should be $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix}$ and therefore, $x_1 = 1$ and $x_2 = 2$.

Exercise 4

Solve the following linear system:
$$\begin{array}{rcrcl} & & -2\,x_2 & = & 6 \\ 2\,x_1 & + & 4\,x_2 & = & -8 \end{array}$$

Follow the instructions below to begin this exercise. Once you have found the solution, fill in the missing values immediately below (to enter a number, click inside each empty box (□) and type in the appropriate number and then press the **TAB** key to quickly move to the next box).

$x_1 = □,$ $x_2 = □$

Check your solution (by hand) by substituting the values of x_1 and x_2 into each of the original equations.

Execute the following assignment statement to set up the corresponding augmented matrix.

```
M = ( 0  -2   6 ) ;
    ( 2   4  -8 )
```

The first step is to switch rows, by executing the following command:

```
Swap[M, 1, 2]
```

Now continue the process to find a solution.

Exercise 5

Solve the following linear system:
$$\begin{array}{rcrcrcrcl} 3\,x_1 & - & 2\,x_2 & + & 6\,x_3 & = & 33 \\ 6\,x_1 & - & 4\,x_2 & + & 5\,x_3 & = & 31 \\ 9\,x_1 & & & + & x_3 & = & 14 \end{array}$$

using the commands **ReplaceRow, Scale** and **Swap.**

Begin by executing the assignment statement below to set up the corresponding augmented matrix. Once you have found the solution, fill in the missing values immediately below:

$x_1 = □,$ $x_2 = □,$ $x_3 = □$

$$M = \begin{pmatrix} 3 & -2 & 6 & 33 \\ 6 & -4 & 5 & 31 \\ 9 & 0 & 1 & 14 \end{pmatrix};$$

Exercise 6

In this exercise, you will use the commands **ReplaceRow**, **Scale** and **Swap** to solve the following linear system

$$
\begin{array}{rcrcrcrcl}
x_1 & & & - & 2\,x_3 & & & = & -1 \\
& & x_2 & & & - & x_4 & = & 2 \\
& & -3\,x_2 & + & 2\,x_3 & & & = & 0 \\
-4\,x_1 & & & & & + & 7\,x_4 & = & -5
\end{array}
$$

Begin by executing the following assignment statement representing the corresponding augmented matrix. Once you have found the solution, fill in the missing values immediately below.

$x_1 = \square,\quad x_2 = \square,\quad x_3 = \square,\quad x_4 = \square$

$$M = \begin{pmatrix} 1 & 0 & -2 & 0 & -1 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & -3 & 2 & 0 & 0 \\ -4 & 0 & 0 & 7 & -5 \end{pmatrix}$$

Using the Symbolic Power of *Mathematica*

Suppose you want to determine the values of h such that $\begin{pmatrix} 1 & h & 2 \\ 2 & -4 & 5 \end{pmatrix}$ is the augmented matrix of a consistent linear system. Execute the following cell. (The **Clear** command is used to clear out any value that may have been assigned to h during the current computer session.)

```
Clear[h]; M = ( 1  h  2 )
             ( 2 -4  5 );
```

Now execute the following command:

```
ReplaceRow[M, 2, -2, 1]
```

From observing the bottom row, the system is inconsistent if and only if $-4 - 2\,h = 0$. So, the system is consistent if and only if $h \neq -2$.

Exercise 7

Determine the value(s) of h such that the following matrix is the augmented matrix of a consistent system.

$$\begin{pmatrix} 1 & 4 & -2 \\ 3 & h & -6 \end{pmatrix}$$

Begin by executing the following cell and when you complete your work, answer the following question (click at the end of the question below and begin typing):

For what values of h is the system consistent?

```
Clear[h]; M = ( 1  4  -2 )
             ( 3  h  -6 )
```

Exercise 8

Find an equation involving g, h, and k that makes the following augmented matrix correspond to a consistent system

$$\begin{pmatrix} 2 & 5 & -3 & g \\ 4 & 7 & -4 & h \\ -6 & -3 & 1 & k \end{pmatrix}$$

Begin by executing the following cell and when you have found the answer, complete the following statement by replacing each box with an appropriate number.

The system is consistent if and only if $\square\, g + \square\, h + \square\, k = \square$.

```
Clear[g, h, k]; M = ( 2   5  -3  g )
                     ( 4   7  -4  h );
                     (-6  -3   1  k )
```

Printing Instructions

Before printing your document, choose **Edit** followed by **Select All** from the menu. Then choose **Format**, then **Word Wrapping** followed by **Wrap at Paper Width**. Then print your document by pressing the printing icon at the top of the notebook.

Initialization Cell

At the end of each computer assignment will be a section called **Initialization Cell.** This section contains commands that will be automatically loaded into memory when the first *Mathematica* command is executed. For example, the command **Needs["LayData`LayFunctions`"]** in the following cell automatically loads some additional commands needed for this assignment. Since commands contained in the initialization cell are automatically executed, you can ignore this cell.

```
Needs["LayData`LayFunctions`"]
```