Solving a Linear System:

3 basic operations (Elementary Row Operations)

1. (Replacement)
   Replace 1 row by the sum of itself and a multiple of another row
2. (Interchange)
   Interchange 2 rows
3. (Scaling)
   Multiply all enties in a row by a nonzero constant.

example:

$$\begin{cases} x_1 + 5x_2 - 3x_3 = 1 \\ 2x_1 - x_2 = 0 \\ 7x_2 + 2x_3 = 4 \end{cases}$$

$$\begin{array}{ll} (\mathrm{I}) & x_1 + 5x_2 - 3x_3 = 1 \\ (\mathrm{II}) & 2x_1 - x_2 = 0 \\ (\mathrm{III}) & 7x_2 + 2x_3 = 4 \end{array} \qquad \begin{bmatrix} 1 & 5 & -3 & 1 \\ 2 & -1 & 0 & 0 \\ 0 & 7 & 2 & 4 \end{bmatrix}$$

keep x_1 in 1^{st} ~~column~~ equation. Eliminate it from all others.

$$\begin{array}{rl} -2(\mathrm{I}): & -2x_1 - 10x_2 + 6x_3 = -2 \\ +(\mathrm{II}): & 2x_1 - x_2 = 0 \\ \hline \text{new } (\mathrm{II}) & -11x_2 + 6x_3 = -2 \end{array}$$

$$\begin{array}{l} x_1 + 5x_2 - 3x_3 = 1 \\ -11x_2 + 6x_3 = -2 \\ 7x_2 + 2x_3 = 4 \end{array} \qquad \begin{bmatrix} 1 & 5 & -3 & 1 \\ 0 & -11 & 6 & -2 \\ 0 & 7 & 2 & 4 \end{bmatrix}$$

want coef of x_2 in 2^{nd} row to be 1

$$
\begin{array}{ll}
(\mathrm{I}) & x_1 + 5x_2 - 3x_3 = 1 \\
(\mathrm{II}) & x_2 - \frac{6}{11} x_3 = \frac{2}{11} \\
(\mathrm{III}) & 7x_2 + 2x_3 = 4
\end{array}
\qquad
\begin{bmatrix} 1 & 5 & -3 & 1 \\ 0 & 1 & -\frac{6}{11} & \frac{2}{11} \\ 0 & 7 & 2 & 4 \end{bmatrix}
$$

eliminate x_2 from all equations except row 2

$$
\begin{array}{rl}
(\mathrm{I}): & x_1 + 5x_2 - 3x_3 = 1 \\
-5(\mathrm{II}): & -5x_2 + \frac{30}{11} x_3 = -\frac{10}{11} \\
\hline
\text{new } (\mathrm{I}) & x_1 - \frac{3}{11} x_3 = \frac{1}{11}
\end{array}
$$

$$
\begin{array}{rl}
(\mathrm{III}): & 7x_2 + 2x_3 = 4 \\
-7(\mathrm{II}): & -7x_2 + \frac{42}{11} x_3 = -\frac{14}{11} \\
\hline
\text{new } (\mathrm{III}) & \frac{64}{11} x_3 = \frac{30}{11}
\end{array}
$$

$$
\begin{array}{l}
x_1 - \frac{3}{11} x_3 = \frac{1}{11} \\
x_2 - \frac{6}{11} x_3 = \frac{2}{11} \\
\frac{64}{11} x_3 = \frac{30}{11}
\end{array}
\qquad
\begin{bmatrix} 1 & 0 & -\frac{3}{11} & \frac{1}{11} \\ 0 & 1 & -\frac{6}{11} & \frac{2}{11} \\ 0 & 0 & \frac{64}{11} & \frac{30}{11} \end{bmatrix}
$$

want coef of x_3 in 3^{rd} row to be 1

$$
\begin{array}{ll}
(\mathrm{I}) & x_1 - \frac{3}{11} x_3 = \frac{1}{11} \\
(\mathrm{II}) & x_2 - \frac{6}{11} x_3 = \frac{2}{11} \\
(\mathrm{III}) & x_3 = \frac{30}{64} = \frac{15}{32}
\end{array}
\qquad
\begin{bmatrix} 1 & 0 & -\frac{3}{11} & \frac{1}{11} \\ 0 & 1 & -\frac{6}{11} & \frac{2}{11} \\ 0 & 0 & 1 & \frac{15}{32} \end{bmatrix}
$$

eliminate x_3 from all equations except row 3

$$
\begin{array}{rl}
(\mathrm{I}): & x_1 - \frac{3}{11} x_3 = \frac{1}{11} = \frac{32}{352} \\
+\frac{3}{11}(\mathrm{III}): & \frac{3}{11} x_3 = \frac{45}{352} \\
\hline
\text{new } (\mathrm{I}) & x_1 = \frac{77}{352} = \frac{7}{32}
\end{array}
$$

(II) :	$x_2 - \frac{6}{11}x_3 = \frac{2}{11} = \frac{32}{176}$	
$+\frac{6}{11}$(III) :	$\frac{6}{11}x_3 = \frac{90}{352} = \frac{45}{176}$	
new (II)	$x_2 = \frac{77}{176} = \frac{7}{16}$	

$$x_1 = \frac{7}{32}$$
$$x_2 = \frac{7}{16}$$
$$x_3 = \frac{15}{32}$$

$$\begin{bmatrix} 1 & 0 & 0 & \frac{7}{32} \\ 0 & 1 & 0 & \frac{7}{16} \\ 0 & 0 & 1 & \frac{15}{32} \end{bmatrix}$$

Check is this a solution? ✓

To Solve this we used <u>Row</u> <u>Operations</u>

* we can apply these row operations to <u>any</u> matrix

Row Equivalent – 2 matrices are row equivalent if there exist a sequence of row operations that transforms 1 matrix into the 2nd matrix

Note: all row operations are <u>reversible</u>

how?

3. Scaling?
2. Interchangeing?
1. Replacement?

the solution sets of 2 row equivalent matrices are the same