if λ is an eigenvalue : $A\bar{x} = \lambda\bar{x}$

$$A\bar{x} - \lambda\bar{x} = 0$$

$$(A - \lambda \cdot I)\bar{x} = 0$$

but this means that

$$\det(A - \lambda I) = 0$$

if $\bar{x} \neq 0$, and this makes finding $\bar{x}$ easier

<u>ex</u>: $A = \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}$ find the eigenvalues

$$\det(A - \lambda I) = \begin{vmatrix} 2-\lambda & 1 \\ -1 & 4-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)(4-\lambda) - (1)(-1) = 0$$

$$8 - 6\lambda + \lambda^2 + 1 = 0$$

$$\underbrace{\lambda^2 - 6\lambda + 9}_{\text{the characteristic equations}} = 0$$

$$(\lambda - 3)(\lambda - 3) = 0$$

$$\lambda = 3$$

here we have 1 distinct eigenvalue
but it has multiplicity 2

$$(\lambda - 3)^2 = 0$$

Characteristic Equation

$$\det(A - \lambda I) = 0$$

λ - eigenvalue

A is nxn matrix

←

Thrm (Invertible matrix thrm):

A is an $n \times n$ matrix

A is invertible $\iff$ 0 is <u>not</u> an eigenvalue

<u>ex</u>: Find the characteristic equation of

$$A = \begin{bmatrix} 2 & 10 & 3 & 9 \\ 0 & 2 & -2 & 5 \\ 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 7 \end{bmatrix}$$

$$\det(A-\lambda I) = \begin{vmatrix} (2-\lambda) & 10 & 3 & 9 \\ 0 & (2-\lambda) & -2 & 5 \\ 0 & 0 & (-4-\lambda) & 1 \\ 0 & 0 & 0 & (7-\lambda) \end{vmatrix}$$

$$= (2-\lambda)(2-\lambda)(-4-\lambda)(7-\lambda)$$

$$\Rightarrow -(2-\lambda)^2(4+\lambda)(7-\lambda) = 0$$

So we have the following eigenvalues:

$\lambda = 2$	multiplicity	2
$\lambda = -4$	"	1
$\lambda = 7$	"	1

<u>ex</u>: a 5×5 matrix has a characteristic equation: $\lambda^5 - 4\lambda^4 + 3\lambda^3 = 0$ find the eigenvalues.

$$\Rightarrow \lambda^3(1-\lambda)(3-\lambda) = 0$$

$$\lambda = 1, 3, 0$$

↑ multiplicity 3

←

Similar Matrices

A is similar to B if, $\exists P$ & $Q \ni$

$$A = PBP^{-1}$$

OR $A = Q^{-1}BQ$

Theorem

if $n \times n$ matrices A & B are similar, then they have the same characteristic equations

$\Rightarrow$ they have the same eigenvalues

Note: if A is row equivalent to B (ie $A \sim B$) then $\exists$ invertible matrix $E \ni A = EB$

the eigenvalues of A & B are usually different.

$\leftarrow$