Example: Is $\mathbb{Z}_3/\langle x^2 + 1 \rangle$ a field? Write out its addition and multiplication tables.

The addition table of the congruence classes:

$\oplus$	$[0]$	$[1]$	$[2]$	$[x]$	$[x+1]$	$[x+2]$	$[2x]$	$[2x+1]$	$[2x+2]$
$[0]$	$[0]$	$[1]$	$[2]$	$[x]$	$[x+1]$	$[x+2]$	$[2x]$	$[2x+1]$	$[2x+2]$
$[1]$	$[1]$	$[2]$	$[0]$	$[x+1]$	$[x+2]$	$[x]$	$[2x+1]$	$[2x+2]$	$[2x]$
$[2]$	$[2]$	$[0]$	$[1]$	$[x+2]$	$[x]$	$[x+1]$	$[2x+2]$	$[2x]$	$[2x+1]$
$[x]$	$[x]$	$[x+1]$	$[x+2]$	$[2x]$	$[2x+1]$	$[2x+2]$	$[0]$	$[1]$	$[2]$
$[x+1]$	$[x+1]$	$[x+2]$	$[x]$	$[2x+1]$	$[2x+2]$	$[2x]$	$[1]$	$[2]$	$[0]$
$[x+2]$	$[x+2]$	$[x]$	$[x+1]$	$[2x+2]$	$[2x]$	$[2x+1]$	$[2]$	$[0]$	$[1]$
$[2x]$	$[2x]$	$[2x+1]$	$[2x+2]$	$[0]$	$[1]$	$[2]$	$[x]$	$[x+1]$	$[x+2]$
$[2x+1]$	$[2x+1]$	$[2x+2]$	$[2x]$	$[1]$	$[2]$	$[0]$	$[x+1]$	$[x+2]$	$[x]$
$[2x+2]$	$[2x+2]$	$[2x]$	$[2x+1]$	$[2]$	$[0]$	$[1]$	$[x+2]$	$[x]$	$[x+1]$

The multiplication table of the congruence classes:

$\odot$	$[0]$	$[1]$	$[2]$	$[x]$	$[x+1]$	$[x+2]$	$[2x]$	$[2x+1]$	$[2x+2]$
$[0]$	$[0]$	$[0]$	$[0]$	$[0]$	$[0]$	$[0]$	$[0]$	$[0]$	$[0]$
$[1]$	$[0]$	$[1]$	$[2]$	$[x]$	$[x+1]$	$[x+2]$	$[2x]$	$[2x+1]$	$[2x+2]$
$[2]$	$[0]$	$[2]$	$[1]$	$[2x]$	$[2x+2]$	$[2x+1]$	$[x]$	$[x+2]$	$[x+1]$
$[x]$	$[0]$	$[x]$	$[2x]$	$[2]$	$[x+2]$	$[2x+2]$	$[1]$	$[x+1]$	$[2x+1]$
$[x+1]$	$[0]$	$[x+1]$	$[2x+2]$	$[x+2]$	$[2x]$	$[1]$	$[2x+1]$	$[2]$	$[x]$
$[x+2]$	$[0]$	$[x+2]$	$[2x+1]$	$[2x+2]$	$[1]$	$[x]$	$[x+1]$	$[2x]$	$[2]$
$[2x]$	$[0]$	$[2x]$	$[x]$	$[1]$	$[2x+1]$	$[x+1]$	$[2]$	$[2x+2]$	$[x+2]$
$[2x+1]$	$[0]$	$[2x+1]$	$[x+2]$	$[x+1]$	$[2]$	$[2x]$	$[2x+2]$	$[x]$	$[1]$
$[2x+2]$	$[0]$	$[2x+2]$	$[x+1]$	$[2x+1]$	$[x]$	$[2]$	$[x+2]$	$[1]$	$[2x]$