The Divisor Theorem: Let b, c and d be integers with $d > 0$.

(1) If b divides c, then b divides $-c$ and $-b$ divides both c and $-c$.
(2) If d divides b and b is positive, then $d \leq b$.
(3) If d divides c and c is negative, then $c \leq -d$ and $d \leq -c$.
(4) If $0 < b \leq d$ and d divides b, then $d = b$.
(5) If d divides b, b divides d and b is positive, then $b = d$.
(6) If $b = dt$ for some integer t and $1 < d < b$, then $1 < t < b$.
(7) If $d = ds$ for some integer s, then $s = 1$.

The GCD Theorem: Let f, g and d be integers with $f^2 + g^2 > 0$ and let $S = \{e \in \mathbb{Z} \mid e = fa + gb > 0$ for some integers $a, b\}$. Then the following are equivalent (if any one is true about f, g and d, all five are true).

(1) d is the smallest integer in the set S.
(2) (i) $d > 0$, (ii) $f = dh$ and $g = di$ for some integers h and i, **AND** (iii) there is a pair of integers s and t such that $d = fs + gt$.
(3) (i) $d > 0$, (ii) d divides both f and g, **AND** (iii) there is a pair of integers k and m such that $d = fk + gm$.
(4) (i) $d > 0$, (ii) d divides both f and g, **AND** (iii) if c is an integer that divides both f and g, then c divides d.
(5) $d = gcd(f, g)$.

GCD = 1 Theorem: Let a and b be integers. Then $gcd(a, b) = 1$ if and only if there are integers f and g such that $af + bg = 1$.

Let $n > 1$ be a positive integer. Then for each integer a, $[a]$ denotes the set $\{b \in \mathbb{Z} \,|\, b \equiv a \pmod{n}\}$.

The Congruence Theorem: Let a, b, and n be integers with $n > 1$. Then the following are equivalent.

(1) $a \equiv b \pmod{n}$.
(2) n divides $a - b$.
(3) There is an integer k such that $a - b = nk$.
(4) There is an integer m such that $a = b + nm$.
(5) There is an integer q such that $b = a + nq$.
(6) There is an integer r such that $b - a = nr$.
(7) n divides $b - a$.
(8) $b \equiv a \pmod{n}$.
(9) There is an integer c such that $a \equiv c \pmod{n}$ and $b \equiv c \pmod{n}$.
(10) There is an integer d such that $a \equiv d \pmod{n}$ and $d \equiv b \pmod{n}$.
(11) There is an integer e such that $e \equiv a \pmod{n}$ and $e \equiv b \pmod{n}$.
(12) There is an integer f such that $f \equiv a \pmod{n}$ and $b \equiv f \pmod{n}$.
(13) $[a] = [b]$.
(14) $[a] \subseteq [b]$.
(15) For each integer h, if $h \in [a]$, then $h \in [b]$.
(16) For each integer s, if $s \in [b]$, then $s \in [a]$.
(17) $[b] \subseteq [a]$.
(18) $[a] \cap [b]$ is nonempty.
(19) $a \in [b]$.
(20) $b \in [a]$.