due March 27 Homework Set 10 Name: _____
(sections 4.3 – 4.4)

When writing a proof, be sure to cite all of the properties, theorems, corollaries, and definitions you use.

1. Find a monic associate of
   a. $3x^5 - 4x^2 + 1$ in $\mathbb{Z}_5[x]$

   b. $ix^3 + x - 1$ in $\mathbb{C}[x]$

2. Determine whether each polynomial is irreducible. If the polynomial is reducible express it as a product of its irreducibles.
   a. $x^2 - 3$ in $\mathbb{Q}[x]$? in $\mathbb{R}[x]$?

   b. $x^2 + x - 2$ in $\mathbb{Z}_3[x]$? In $\mathbb{Z}_7[x]$?

   c. $x^4 - 4$ in $\mathbb{Q}[x]$? in $\mathbb{R}[x]$? in $\mathbb{C}[x]$?

3. Without doing any polynomial multiplication, use the Factor Theorem to show that $x^7 - x$ factors in $\mathbb{Z}_7[x]$ as $x(x-1)(x-2)(x-3)(x-4)(x-5)(x-6)$.

4. Prove that $f(x)$ is irreducible in $F[x]$ if and only if each of its associates is irreducible.

5. Prove Theorem 4.10
   Theorem 4.10: Let F be a field. A nonzero polynomial $f(x)$ is reducible in $F[x]$ if and only if $f(x)$ can be written as the product of two polynomials of lower degree.

6. Read Appendix C. Explain the difference between regular mathematical induction and complete mathematical induction.