

due April 10

Homework Set 12
(Sections 5.1 – 5.2)

Name: _____

When writing a proof, be sure to cite all of the properties, theorems, corollaries, and definitions you use. Be sure to write all of your answers in complete sentences (even the non-proof questions).

1. Let $f(x), g(x)$, and $p(x)$ be in $\mathbb{Q}[x]$. Determine whether $f(x) \equiv g(x) \pmod{p(x)}$ where $f(x) = x^4 + 2x^3 - 3x^2 + x - 5$, $g(x) = x^4 + x^3 - 5x^2 + 12x - 25$, and $p(x) = x^2 - 2x + 4$

2. Explain why $[2x - 3]$ is a unit in $\mathbb{Q}[x]/\langle x^2 - 2 \rangle$, and find its inverse.

3. Let a be in a field F . Describe the congruence classes in $F[x]$ modulo the polynomial $x - a$.

4. Write out the addition and multiplication tables for $\mathbb{Z}_5[x]/\langle x^2 + 1 \rangle$. Is $\mathbb{Z}_5/\langle x^2 + 1 \rangle$ a field? Why or why not? (Note: these tables will be quite large. Feel free to write them out on a separate sheet of paper.)

5. Prove that $\mathbb{Q}[x]/\langle x^2 \rangle$ is a field, or give a reason why it is not a field. (Hint: Each element of $\mathbb{Q}[x]/\langle x^2 \rangle$ can be written in the form $[ax + b]$. Be sure to state why this is true if you use this fact.)