due April 17 | Homework Set 13 (Section 5.3) | Name: _____________

When writing a proof, be sure to cite all of the properties, theorems, corollaries, and definitions you use. Be sure to write all of your answers in complete sentences (even the non-proof questions).

1. Factor $3x^6 + 2x^5 + 2x^4 + x^3 + x^2 + 4x + 2$ into its product of irreducibles in $\mathbb{Z}_5[x]$.

2. Determine whether the given congruence-class ring is a field:

   a. $\mathbb{Z}_3[x]/\langle x^3 + 2x^2 + x + 1\rangle$

   b. $\mathbb{Z}_5[x]/\langle 2x^3 - 4x^2 + 2x + 1\rangle$

3. Let F be a field. If a is an element of F, describe the field $F[x]/\langle x-a\rangle$. (ie: what do the elements of $F[x]/\langle x-a\rangle$ look like; how do the two fields F and $F[x]/\langle x-a\rangle$ compare, etc.)

4. Suppose K is a ring that contains $\mathbb{Z}_6$ as a subring. Show that $3x^2+1$ in $\mathbb{Z}_6[x]$ has no roots in K. Why does Corollary 5.12 fail in this situation?

5. Let F be a field and $p(x)$ irreducible in $F[x]$. If $f(x) \neq 0_F$ in $F[x]/\langle p(x)\rangle$ and $h(x)$ is in $F[x]$, prove that there exists a polynomial $g(x)$ in $F[x]$ such that $f(x)g(x) = h(x)$ in $F[x]/\langle p(x)\rangle$. [Note: the polynomials in $F[x]/\langle p(x)\rangle$ are actually equivalence classes.]