due April 24 | Homework Set 14 (Section 6.1) | Name: _____________

When writing a proof, be sure to cite all of the properties, theorems, corollaries, and definitions you use. Be sure to write all of your answers in complete sentences (even the non-proof questions).

1. Consider the following ideals in $\mathbb{Z}$:
   a. Find principal ideals $\langle a \rangle$ and $\langle b \rangle$ in the ring $\mathbb{Z}$ such that $\langle a \rangle = \langle b \rangle$ but that $a \neq b$.

   b. Show that $\langle 4,6 \rangle = \langle 2 \rangle$ in $\mathbb{Z}$.

2. Let I and J be ideals in ring R. Prove that the set $K = \{a + b \mid a \in I, b \in J\}$ is an ideal in R that contains both I and J. Note, K is called the sum of I and J and is denoted by $I + J$.

3. Verify that $I = \{0,3,6,9,12\}$ is an ideal in $\mathbb{Z}_{15}$ and list all of its distinct cosets.

4. Let $I = \{0,5\}$ in $\mathbb{Z}_{10}$. Verify that I is an ideal and show that $\mathbb{Z}_{10}/I \cong \mathbb{Z}_5$.

5. List all principal ideals in $\mathbb{Z}_{12}$. Then write out the addition and multiplication tables of $\mathbb{Z}_{12}/I$ where $I = \langle 3 \rangle$.