due May 1 | Homework Set 15 (Section 6.2) | Name: _____________

When writing a proof, be sure to cite all of the properties, theorems, corollaries, and definitions you use. Be sure to write all of your answers in complete sentences (even the non-proof questions).

1. Let $[a]_n$ denote the equivalence class of a modulo n.
    a. Show that the map $f: \mathbb{Z}_{12} \to \mathbb{Z}_4$ that sends $[a]_{12}$ into $[a]_4$ is a surjective homomorphism.

    b. What is the kernel of f?

2. Show that $\mathbb{Z}_{20}/\langle 5 \rangle \cong \mathbb{Z}_5$.

3. Let $f: R \to S$ be a surjective ring homomorphism, and let I be an ideal of R. Prove that $f(I)$ is an ideal of S, where $f(I) = \{s \in S \mid s = f(a) \ for \ some \ a \in I\}$.

4. Let I be an ideal in a non-commutative ring R such that $ab - ba \in I$ for all $a, b \in I$. Prove that R/I is commutative.