due Feb 7 | Homework Set 4 (section 2.1 – 2.3) | Name: _____________

1. Let a and b be integers with n and k positive integers such that if $a \equiv b \ \ mod \ \ n$ and $k|n$, prove that $a \equiv b \ \ mod \ \ k$.

2. Prove part (8) of Theorem 2.7. (Be sure to give the statement of the theorem.)

3. Let a, b, n be integers with $n > 1$. Let $d = \gcd(a, n)$. If $[a]x = [b]$ has a solution in $\mathbb{Z}_n$, prove that $d|b$. [Hint: if $x = [r]$ is a solution, then $[ar] = [b]$ so that $ar - b = nk$ for some integer k.]

4. Write out the addition and multiplication tables for $\mathbb{Z}_{12}$.

5. Answer the following computational questions. Show all necessary work.
    a. Find an element of $\mathbb{Z}_7$ such that every non-zero element of $\mathbb{Z}_7$ is a positive power of that element.

    b. Solve the equation: $x^2 + 1 = 0$ in $\mathbb{Z}_{12}$. You may assume that $0 \leq x < 12$.