due Feb 21 Homework Set 6 Name: _____
(section 2.3 & 3.1)

When writing a proof, be sure to cite all of the properties, theorems, corollaries, and definitions you use. Note, questions 1 and 2 will require a proof.

1. Show that $\mathbb{Z}[\sqrt{5}] = \{a + b\sqrt{5} \mid a, b \in \mathbb{Z}\}$ is a subring of $\mathbb{R}$.

2. Show that $\{0, 2, 4, 6, 8\}$ is a subring of $\mathbb{Z}_{10}$. Does this subring have an identity element (ie: a multiplicative identity)? If so, what is it?

3. Solve the following equations for x. If no such x exists, explain why.

   a. $7x \equiv 11 \; mod \; 450$ $\qquad x \equiv$ _____ $mod \; 450$

   b. $6x \equiv 3 \; mod \; 16$ $\qquad x \equiv$ _____ $mod \; 16$

   c. $15x \equiv 9 \; mod \; 18$ $\qquad x \equiv$ _____ $mod \; 18$

4. Write out the addition and multiplication tables for $\mathbb{Z}_2 \times \mathbb{Z}_3$.