due Feb 28 Homework Set 7 (section 3.2) Name: _____

When writing a proof, be sure to cite all of the properties, theorems, corollaries, and definitions you use. Also, be sure that you do not skip steps and that you define all of the terms you use.

1. Prove parts (4) and (6) of Theorem 3.5.
   Theorem 3.5: For any elements a and b of a ring R, the following properties are true:
   (4) $-(a+b) = (-a) + (-b)$
   (6) $(-a)(-b) = ab$

2. Let S and T be subrings of a ring R. Is $S \cap T$ a subring of R? Either prove that it is a subring, or give a counter-example where $S \cap T$ is not a subring.

3. An element a is called *idempotent* if $a^2 = a$. Find all idempotent elements in $\mathbb{Z}_{12}$.

4. In the ring $M_{2\times 2}(\mathbb{R})$, let

$$A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 4 & 1 \\ 1 & -1 \end{pmatrix} \quad C = \begin{pmatrix} 2 & 3 \\ 1 & -1 \end{pmatrix} \quad D = \begin{pmatrix} 4 & -7 \\ 1 & -3 \end{pmatrix}$$

Show that the equations $AX = B$ and $XA = B$ have different solutions.

5. Is it possible for a unit in a ring with identity to be a zero divisor? Prove your answer.