due March 20 | Homework Set 9 (sections 4.1 – 4.2) | Name: _____________

When writing a proof, be sure to cite all of the properties, theorems, corollaries, and definitions you use.

1. Give an example of polynomials $f(x), g(x) \in \mathbb{Q}[x]$ that satisfy each given condition:

   a. $\deg[f(x) + g(x)] < \max\{f(x), g(x)\}$

   b. $\deg[f(x) + g(x)] = \max\{f(x), g(x)\}$

2. Let R be a ring. If $f(x), g(x) \in R[x]$ and $f(x) + g(x) \neq 0_R$, prove that $\deg[f(x) + g(x)] \leq \max\{f(x), g(x)\}$

3. Let $D: \mathbb{R}[x] \to \mathbb{R}[x]$ be the derivative map defined by $D(a_0 + a_1x + a_2x^2 \ldots + a_nx^n) = a_1 + 2a_2x + \cdots + na_nx^{n-1}$. Is D a homomorphism of rings? An isomorphism?

4. Let F be a field and $f(x), g(x) \in F[x]$. If $f(x)$ divides $g(x)$ and (x) divides $f(x)$, show that $f(x) = cg(x)$ for some non-zero $c \in F$.

5. Use the Euclidean Algorithm to find the gcd of $f(x) = 4x^4 + 2x^3 + 6x^2 + 4x + 5$ and $g(x) = 3x^3 + 5x^2 + 6x$ in $\mathbb{Z}_7[x]$.