We won't be doing proofs like the below until late February.

Here is the basic scheme for proving a set X is a "subring" of some given ring Y.

Generally the set X will be given in the form $X = \{s \in Y \mid s \text{ "does something special"}\}$ where "does something special" will vary from problem to problem–**don't look back and try to use the same "does something special" from a previous problem**. You have to use what "does something special" is for the particular problem you are working on.

The conclusion will always have the form "Therefore X is subring of Y by the Subring Theorem." where X is the name of the set you are supposed to prove is a subring of the ring whose name is Y.

Step 1: Show the zero element of the ring Y satisfies the "does something special" for the set X. Generally this will require a theorem. Possibilities for which particular theorem is needed include the "Ring Theorem" and the "Homomorphism Theorem". The zero element of the ring Y is usually written as 0_Y, unless we know more about the ring Y. For example the zero element of the ring $\mathbb{Z}_n$ is written as $[0]$. Once you have established that the zero element of the ring Y satisfies the "does something special", then the next sentence would be, "Thus 0_Y is in X." or the condensed version "Thus $0_Y \in X$."

Step 2: Next pick two letters to represent a pair of elements of the set X. The sentence should simply say "Let $b, c \in X$." or "Let b and c be elements of X." **DO NOT SAY ANYTHING ELSE IN THIS SENTENCE–YOU WILL BE PENALIZED IF YOU DO.** To be safe, do not use letters that appear anywhere in the statement of the problem because there will be problems where the "does something special" involves some particular fixed element of the ring Y. For example, if the problem says the letter t is a fixed element of Y, then you cannot use t to represent an element of X (and with regard to the warning in Step 3, you cannot use it for the "for some ...").

Step 3: The sentence after "Let $b, c \in X$" should be where you say that b and c satisfy the "does something special" condition. Usually some kind of equation will be part of the "does something special". If so, put b and c in the same place as the letter on the **left side** of the "$|$" in the description of the set $X = \{s \in Y \mid s \text{ "does something special"}\}$, and include any phrase that might be included in the "does something special" – in this example b and c would go where the "s" is on the **right side** of the "$|$". Warning: if the "does something special" includes a "for some ....", you must use different letters for the "for some ...", one for each of b and c.

Step 4: The actual "proof part" is what comes next. You have to show $b + c$, $-b$ and bc are also in the set X by showing they also satisfy the "does something special". What you do here varies from problem to problem. Frequently the sum will use the distributive property of multiplication over addition–but sometimes it does not. Almost always you will need a theorem to get $-b$ to satisfy the "does something special". The two that will be used most often are the "Ring Theorem" and the "Homomorphism Theorem." Sometimes you will need a theorem ("Ring Theorem" or "Homomorphism Theorem") to show the product bc satisfies the "does something special", but sometimes the only thing you need is the associative property of multiplication. You must say that all three of $b + c$, $-b$ and bc are in the set X after showing that each satisfies the "does something special".

Step 5: Finally end with "Therefore X is a subring of Y by the Subring Theorem."