Due April 4 Homework Set 10 Name:_____
(Sections 4.1 – 4.3)

Write all answers or solutions in complete, grammatically correct sentences. Be sure to cite all reasons.

1. Find an associate of
   a. $x^2 + x + 1$ in $\mathbb{Z}_5[x]$

   b. $3x + 2$ in $\mathbb{Z}_7[x]$

2. Determine whether each polynomial is irreducible. If the polynomial is reducible express it as a product of its irreducibles.
   a. $x^2 - 3$ in $\mathbb{Q}[x]$? in $\mathbb{R}[x]$?

   b. $x^2 + x - 2$ in $\mathbb{Z}_3[x]$? In $\mathbb{Z}_7[x]$?

   c. $x^4 - 16$ in $\mathbb{Q}[x]$? in $\mathbb{R}[x]$? in $\mathbb{C}[x]$?

3. If R is an integral domain, either prove or disprove that $R[x]$ is an integral domain.

4. Let R be a ring. If $f(x), g(x) \in R[x]$ and $f(x) + g(x) \neq 0_R$, prove that $\deg[f(x) + g(x)] \leq \max\{\deg(f(x)), \deg(g(x))\}$.

5. Use the Euclidean Algorithm to find the greatest common divisor of $f(x) = x^4 + 3x^3 + 2x + 4$ and $g(x) = x^2 - 1$ in $\mathbb{Z}_5[x]$.