due April 18 Homework Set 12 (section 5.1 – 5.2) Name: _____

When writing a proof, be sure to cite all of the properties, theorems, corollaries, and definitions you use. Be sure to write all of your answers in complete sentences (even the non-proof questions).

1. Prove that $x^2 + 1$ is reducible in $\mathbb{Z}_p[x]$ if and only if there exists integers a and b such that $p = a + b$ and $ab \equiv 1 \; mod \; p$.

2. Prove or disprove: If $p(x)$ is relatively prime to $k(x)$ and $f(x)k(x) \equiv g(x)k(x) \; mod \; p(x)$, then $f(x) \equiv g(x) \; mod \; p(x)$.

3. Answer the following two questions:
   a. List all of the monic irreducible polynomials of degree 2 in $\mathbb{Z}_3[x]$.

   b. Is $[2x - 3]$ a unit in $\mathbb{Q}[x]/\langle x^2 - 2\rangle$? If so, find its inverse.

4. Let $f(x), g(x)$, and $p(x)$ be in $\mathbb{Q}[x]$. Determine whether $f(x) \equiv g(x)\ mod\ p(x)$.

   a. $f(x) = x^5 - 2x^4 + 4x^3 - 3x + 1;\quad g(x) = 3x^4 + 2x^3 - 5x^2 + 2;$
   $p(x) = x^2 + 1$

   b. $f(x) = x^5 + 4x^4 - x^3 + 13x^2 + 20x + 3;\ \ g(x) = x^3 - x^2 - 7x - 2;$
   $p(x) = x^2 + 5x + 1$

5. Write out the multiplication and addition tables for each ring given below. Is the ring a field?

   a. $\mathbb{Z}_2/\langle x^2 + 1\rangle$

   b. $\mathbb{Z}_3/\langle x^2 + 1\rangle$