due Feb 14 Homework Set 5 Name: _____
(section 2.1 – 2.3)

1. Let a, b, and $n > 1$ be integers. If $[a]x = [b]$ has a solution in $\mathbb{Z}_n$, prove that the $\gcd(a, n)$ divides b. (Hint: if $x = [r]$ is a solution, then $[ar] = [b]$ implying that $ar - b = nk$ for some integer k. Be sure to give the reasons why all of these statements are true.)

2. Let a, b, and $n > 0$ be integers. If $ab = 0$ in $\mathbb{Z}_n$, then either prove or disprove that either $a = 0$ or $b = 0$. What about in $\mathbb{Z}_p$ where p is prime?

3. Write out the addition and multiplication tables for $\mathbb{Z}_8$.

4. Let a, b, and n be positive integers. Prove that $a \equiv b \; mod \; n$ if and only if a and b leave the same remainder when divided by n.

5. Answer the following computational questions. Show all necessary work.
   a. Compute: $(x+2)(x+3)$ in $\mathbb{Z}_5$

   b. Compute: $(x+2)^5$ in $\mathbb{Z}_5$

   c. For which a does $ax = 1$ have a solution in $\mathbb{Z}_8$? You may assume that $0 \leq a < 8$.