

Except for the last question, these questions are computational. Be sure to answer each question in complete sentences if applicable. Cite any theorems or definitions used if applicable.

1. (20 points) Compute the addition and multiplication tables for \mathbb{Z}_{12} .

2. (10 points each) Solve the following equations for x . There may be multiple answers for x , or there may be no solution. If there is an answer, give x such that $0 \leq x < n$ or x in \mathbb{Z}_n . If there is no solution, state why (ie: cite the appropriate theorem).

a. $7x \equiv 1 \pmod{12}$

$$x \equiv \underline{\hspace{2cm}} \pmod{12}$$

b. $8x \equiv 5 \pmod{12}$

$$x \equiv \underline{\hspace{2cm}} \pmod{12}$$

c. $[5] + x = [2]$ in \mathbb{Z}_7

$$x \equiv \underline{\hspace{2cm}} \text{ in } \mathbb{Z}_7$$

d. $[3]x + [6] = [0]$ in \mathbb{Z}_7

$$x \equiv \underline{\hspace{2cm}} \text{ in } \mathbb{Z}_7$$

3. (20 points) Solve $15x \equiv 17 \pmod{97}$ for x . You may assume that $0 \leq x < 97$.

4. (20 points) Verify that $5\mathbb{Z} = \{5n \mid n \in \mathbb{Z}\}$ is a ring using the 8 properties from the definition of a ring.