Except for the last question, these questions are computational. Be sure to answer each question in complete sentences if applicable. Cite any theorems or definitions used if applicable.

1. (20 points) Compute the addition and multiplication tables for $\mathbb{Z}_{12}$.

+	0	1	2	3	4	5	6	7	8	9	10	11
0	0	1	2	3	4	5	6	7	8	9	10	11
1	1	2	3	4	5	6	7	8	9	10	11	0
2	2	3	4	5	6	7	8	9	10	11	0	1
3	3	4	5	6	7	8	9	10	11	0	1	2
4	4	5	6	7	8	9	10	11	0	1	2	3
5	5	6	7	8	9	10	11	0	1	2	3	4
6	6	7	8	9	10	11	0	1	2	3	4	5
7	7	8	9	10	11	0	1	2	3	4	5	6
8	8	9	10	11	0	1	2	3	4	5	6	7
9	9	10	11	0	1	2	3	4	5	6	7	8
10	10	11	0	1	2	3	4	5	6	7	8	9
11	11	0	1	2	3	4	5	6	7	8	9	10

·	0	1	2	3	4	5	6	7	8	9	10	11
0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11
2	0	2	4	6	8	10	0	2	4	6	8	10
3	0	3	6	9	0	3	6	9	0	3	6	9
4	0	4	8	0	4	8	0	4	8	0	4	8
5	0	5	10	3	8	1	6	11	4	9	2	7
6	0	6	0	6	0	6	0	6	0	6	0	6
7	0	7	2	9	4	11	6	1	8	3	10	5
8	0	8	4	0	8	4	0	8	4	0	8	4
9	0	9	6	3	0	9	6	3	0	9	6	3
10	0	10	8	6	4	2	0	10	8	6	4	2
11	0	11	10	9	8	7	6	5	4	3	2	1

2. (10 points each) Solve the following equations for x. There may be multiple answers for x, or there may be no solution. If there is an answer, give x such that $0 \leq x < n$ or x in $\mathbb{Z}_n$. If there is no solution, state why (ie: cite the appropriate theorem).

   a. $7x \equiv 1 \; mod \; 12$

      $\gcd(7,12)=1$

      $7(-5)+12(3)=1$

      $-5 \equiv 7 \bmod 12$

      $x \equiv$ 7 $mod \; 12$

   b. $8x \equiv 5 \; mod \; 12$

      $\gcd(8,12)=4$

      $x \equiv$ no solution ~~mod 12~~

      By Theorem 2.11, there is no solution since gcd(8,12) does not divide 5.

      3pts

   c. $[5] + x = [2]$ in $\mathbb{Z}_7$

      $[5]+[4]=[9]=[2]$

      $x \equiv$ [4] in $\mathbb{Z}_7$

   d. $[3]x + [6] = [0]$ in $\mathbb{Z}_7$

      $x \equiv$ [5] in $\mathbb{Z}_7$

      $[3]x = [1]$ in $\mathbb{Z}_7$

      $\gcd(3,7)=1$

      $3(-2)+7(1)=1$

      $[-2]=[5]$

3. (20 points) Solve $15x \equiv 17 \; mod \; 97$ for x. You may assume that $0 \leq x < 97$.

The $\gcd(15, 97) = 1$, and $1 = 15(13) + 97(-2)$.

Then $15(13) = 1 + 97 \cdot 2 \equiv 1 \bmod 97$. Thus, $15(13 \cdot 17) \equiv 17 \bmod 97$.

Now, $13 \cdot 17 = 221 \equiv 27 \bmod 97$.

Thus, $x \equiv 27 \bmod 97$.

4. (20 points) Verify that $5\mathbb{Z} = \{5n \mid n \in \mathbb{Z}\}$ is a ring using the 8 properties from the definition of a ring.

(2) $\Big\{$ Since $0 = 5 \cdot 0 \in 5\mathbb{Z}$, $5\mathbb{Z}$ is nonempty.
Let $a, b, c \in 5\mathbb{Z}$. Then $a = 5t$, $b = 5r$, $c = 5k$ for some $t, r, k \in \mathbb{Z}$.

2 pt each

1. $a + b = 5t + 5r = 5(t + r) \in 5\mathbb{Z}$.
2. $a + (b + c) = (a + b) + c$ since $5\mathbb{Z}$ is a subset of $\mathbb{Z}$.
3. $a + b = b + a$ since $5\mathbb{Z}$ is a subset of $\mathbb{Z}$.
4. Since $0 \in 5\mathbb{Z}$, it will be the zero element.
5. $x = 5(-t)$ is the solution to $a + x = 0$, and this x is in $5\mathbb{Z}$.
6. $ab = (5t)(5r) = 5(5tr) \in \mathbb{Z}$.
7. $5\mathbb{Z}$ is associative since $\mathbb{Z}$ is.
8. The distributive laws hold since $5\mathbb{Z}$ is a subset of $\mathbb{Z}$.

(2) $\big\{$ Since $5\mathbb{Z}$ satisfies all 8 properties, $5\mathbb{Z}$ is a ring.