due Feb 28 | Homework Set 7 (section 2.3 – 3.1) | Name: _____________

1. Let a be a nonzero element of $\mathbb{Z}_n$. Prove that $ax = 0$ has a nonzero solution in $\mathbb{Z}_n$ if and only if $ax = 1$ has no solution in $\mathbb{Z}_n$. (Hint: you may want to convert everything into $mod\ n$.)

2. Show that the subset $S = \{0,2,4,6,8\}$ of $\mathbb{Z}_{10}$ is a subring. Does S have an identity element?

3. Define a new addition and multiplication on $\mathbb{Z}$ by the rules: $a \oplus b = a + b - 1$ and $a \odot b = ab - (a + b) + 2$. Prove that with these new binary operations $\mathbb{Z}$ is an integral domain. You may assume that under these new operations $\mathbb{Z}$ is a ring.

4. Let $R = \{0, e, b, c\}$ with addition and multiplication defined by the following tables. Assume associativity and distributivity and show that R is a ring with identity. Is R commutative? Is R a field?

$\oplus$	**0**	***e***	***b***	***c***
0	0	e	b	c
e	e	0	c	b
b	b	c	0	e
c	c	b	e	0

$\odot$	**0**	***e***	***b***	***c***
0	0	0	0	0
e	0	e	b	c
b	0	b	b	0
c	0	c	0	c

5. Read Appendix B. Then give an example of a function which is injective but not surjective. Given an example of a second function which is surjective but not injective.