

Be sure to clearly write all of your answers in complete sentences.

- State the definitions for each of the following types of rings. Give an example of each type of ring listed below. (Preference is given for examples which are not any other type of ring.)
  - Ring
  - Ring with identity
  - Commutative Ring
  - Integral Domain
  - Field
  - Polynomial Ring
  - Quotient Ring
  - Subring
  - Ideal
  - Prime Ideal
  - Maximal Ideal
- Determine what type of ring(s) the following sets are. Note: a set may be more than one type of ring. If so, list every type of ring that is it. A set may also not be a ring at all. If so, state which ring axiom fails.
  - $4\mathbb{Z} = \{4n | n \in \mathbb{Z}\}$
  - $\mathbb{Z}_{37}$
  - $\mathbb{Z}_{18}[x]$
  - $\mathbb{Z}[i]$
  - $\left\{ \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \mid a, b \in \mathbb{Z} \right\}$
  - $M_{2 \times 2}(\mathbb{Z}_6)$ , the set of all  $2 \times 2$  matrices with entries in  $\mathbb{Z}_6$
  - Define a new addition and multiplication on  $\mathbb{Q}$  by  $r \oplus s = r + s - 1$  and  $r \odot s = r + s - rs$
  - $\mathbb{Z}_3 \times \mathbb{Z}_4$
  - $\mathbb{Z}_2 \times \mathbb{Z}_2$
  - $\mathbb{Z}_5[x]/\langle x^2 + 4 \rangle$
  - $\mathbb{Q}[x]/\langle x^2 + 1 \rangle$
  - $\mathbb{Q}[x]/\langle x^2 \rangle$
- State the definitions of the following types of elements. Give an example of each type of element. Be sure to clearly state the ring in which your element lives.
  - Zero divisor
  - Unit
  - Associate
  - Identity
  - Reducible
  - Irreducible
  - Monic Polynomial
- Answer the following questions about the degree of a polynomial:
  - What is the definition of the degree of a polynomial?
  - How is the degree of a product and a sum of two polynomials related to the original two polynomials? Give examples. (If the relation is an inequality, be sure to give an example where the equals holds as well as one where the equals does not hold.)
  - What is the degree of the zero polynomial?
- State the definitions of the following types of functions. Give an example of each type of function. Be sure to clearly state the rule, the domain, and the image of the function (ie: what are the rings  $R$  and  $S$  of  $f: R \rightarrow S$ ).
  - Injective function
  - Surjective function
  - Bijjective function
  - Homomorphism
  - Isomorphism
  - Polynomial Function