Answers to Even-Numbered Exercises

Section 1.1

2 $(x_1, x_2) = (12, -7)$.

8 $(0, 0, 0)$.

10 $(-3, -5, 6, -3)$.

14 $(2, -1, 1)$.

16 Consistent.

22 $h = -5/3$.

Section 1.2

10 $x_1 = -4 + 2x_2$, x_2 is free, $x_3 = -7$.

12 $x_1 = 5 + 7x_2 - 6x_4$, x_2 is free, $x_3 = -3 + 2x_4$, x_4 is free,

14 $x_1 = -9 - 7x_3, x_2 = 2 + 6x_3 + 3x_4$, x_3 is free, x_4 is free, $x_5 = 0$.

18 $h \neq -15$.

20 a Inconsistent when $h = 9$ and $k \neq 6$

20 b Unique solution when $h \neq 9$.

20 c Many solutions when $h = 9$ and $k = 6$.

24 The system has no solution.

28 Every column in the augmented matrix except the rightmost column is a pivot column and the rightmost column is not a pivot column.

Section 1.3

6 $-2x_1 + 8x_2 + x_3 = 0, \quad 3x_1 + 5x_2 - 6x_3 = 0$

12 No

14 Yes

Section 1.4

2 Not defined

4 $\begin{bmatrix} 7 \\ 8 \end{bmatrix}$

10 $\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

14 No.

22 Yes

Section 1.5

6 $\mathbf{x} = x_3 \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix}$

14 $\mathbf{x} = \begin{bmatrix} 0 \\ 8 \\ 2 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 3 \\ 1 \\ -5 \\ 1 \end{bmatrix}$

16 $\mathbf{x} = \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix}$

30 a) Yes, b) No

32 a) Yes, b) Yes

Section 1.7

4 LI

8 LD

14 $h = -10$

16 LD

18 LD

20 LD

30 a) n, b) The columns of A are linearly independent if and only if $A\mathbf{x} = 0$ has only the trivial solution. This happens if and only if $A\mathbf{x} = 0$ has no free variables.

32 $\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

34 True

36 False

Section 1.8

2 $\mathbf{x} = \begin{bmatrix} 0.5 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 0.5a \\ 0.5b \\ 0.5c \end{bmatrix}$

Section 2.1

4 $A - 5I_3 = \begin{bmatrix} 4 & -1 & 3 \\ -8 & 2 & -6 \\ -4 & 1 & 3 \end{bmatrix}, \quad (5I_3)A = \begin{bmatrix} 45 & -5 & 15 \\ -40 & 35 & -30 \\ -20 & 5 & 40 \end{bmatrix}$

10 $AB = AC = \begin{bmatrix} 1 & -7 \\ -2 & 14 \end{bmatrix}$

15 a. F; b. F; c. T; d. T; e. F.

16 a. F; b. T; c. F; d. F; e. T.

Section 2.2

6 $x_1 = 2$ and $x_2 = -5$.

9 a. T; b. F; c. F; d. T; e. T.

10 a. F; b. T; c. T; d. T; e. F.

14 Proof.

18 Proof.

32 Not invertible.

Section 2.3

4 LD, not invertible.

11 a. T; b. T; c. F; d. T; e. T.

16 No.

18 Span → Invertible → Unique solution.

34 The standard matrix of T^{-1} is $\frac{1}{2}\begin{bmatrix} 7 & 8 \\ 5 & 6 \end{bmatrix}$

Section 2.4

4 $\begin{bmatrix} A & B \\ -XA+C & -XB+D \end{bmatrix}$

10 $X = -A + BC, Y = -B, Z = -C$

11 a. T; b. F.

Section 2.5

4 $\mathbf{y} = \begin{bmatrix} 0 \\ -5 \\ -18 \end{bmatrix}$ and $\mathbf{x} = \begin{bmatrix} -5 \\ 1 \\ 3 \end{bmatrix}$.

10 $\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & -5 & 1 \end{bmatrix} \begin{bmatrix} -5 & 3 & 4 \\ 0 & -2 & -1 \\ 0 & 0 & 9 \end{bmatrix}$

16 $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 & 0 \\ 3/2 & -2 & 1 & 0 & 0 \\ -3 & 2 & 0 & 1 & 0 \\ 4 & -3 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -6 & 6 \\ 2 & -7 & 5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Section 2.6

4 $\begin{bmatrix} 73.33 \\ 50.00 \\ 30.00 \end{bmatrix}$.

Section 2.7

2 $\begin{bmatrix} -5 & -2 & -4 \\ 0 & 2 & 3 \end{bmatrix}$

4 $\begin{bmatrix} .8 & 0 & -1.6 \\ 0 & 1.2 & 3.6 \\ 0 & 0 & 1 \end{bmatrix}$

6 $\begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 \\ -1/2 & -\sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

8 $\begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 3+2\sqrt{2} \\ \sqrt{2}/2 & \sqrt{2}/2 & 7-5\sqrt{2} \\ 0 & 0 & 1 \end{bmatrix}$

20 The triangle with vertices $(6, 2, 0), (15, 10, 0), (2, 3, 0)$

Section 3.1

2 2

10 -6. Start with row 2.

12 36. Start with row 1 or column 4.

16 2

28 k

32 k

Section 3.2

2 A constant may be factored out of one row.

4 A row replacement operation does not change the determinant.

8 0

12 114

20 7

27 a) T; b) F; c) T; d) F.

34 Proof.

40 a. -2; b. 32; c. -16; d. 1; e. -1.

Section 3.3

4 $\begin{bmatrix} -3/2 \\ 1/2 \end{bmatrix}$.

10 $s \neq 0, 1/4; \quad x_1 = \dfrac{6s-2}{3s(4s-1)}, \quad x_2 = \dfrac{1}{3(4s-1)}$

14 adj $A = \begin{bmatrix} 5 & -3 & -8 \\ 2 & -2 & -3 \\ -4 & 3 & 6 \end{bmatrix}, \quad A^{-1} = (-1)\begin{bmatrix} 5 & -3 & -8 \\ 2 & -2 & -3 \\ -4 & 3 & 6 \end{bmatrix}$

Section 4.1

2 a). Yes. b). It is not a subspace.

6 No.

12 $W = \text{Span}\{\mathbf{u}, \mathbf{v}\}$, where $\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 3 \\ -1 \\ -1 \\ 4 \end{bmatrix}$.

18 $S = \left\{ \begin{bmatrix} 4 \\ 0 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$.

23 c). F, d). T

24 a). T; b). T.

Section 4.2

6 $\begin{bmatrix} -6 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 8 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$

8 No.

14 Yes.

16 $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 5 & -4 \\ 0 & 0 & 1 \end{bmatrix}$

22 $\begin{bmatrix} 7 \\ -4 \\ 1 \\ 0 \end{bmatrix}$ in Nul A, $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ in Col A.

24 In both Nul A and Col A.

25 a). T; b). F; d). F if $A\mathbf{x} = \mathbf{b}$ is not consistent for any $\mathbf{b} \in \mathbf{R^m}$; f). T.

Section 4.3

6 No.

10 $\begin{bmatrix} 5 \\ 4 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ -6 \\ 0 \\ 3 \\ 1 \end{bmatrix}.$

14 Basis for Nul A: $\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ 7/5 \\ 1 \\ 0 \end{bmatrix}$. Basis for Col A: $\begin{bmatrix} 1 \\ 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -5 \\ -5 \\ 0 \\ -5 \end{bmatrix}' \begin{bmatrix} -3 \\ 2 \\ 5 \\ -2 \end{bmatrix}.$

16 $[\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}]$.

20 The three simplest answers are $[\mathbf{v_1}, \mathbf{v_2}]$ or $[\mathbf{v_1}, \mathbf{v_3}]$ or $[\mathbf{v_2}, \mathbf{v_3}]$.

22 a). F; b). T; c). T; d). F; e). F.

Section 4.4

4 $\begin{bmatrix} 0 \\ 1 \\ -5 \end{bmatrix}$.

6 $\begin{bmatrix} -6 \\ 2 \end{bmatrix}$.

10 $\begin{bmatrix} 3 & 2 & 8 \\ -1 & 0 & -2 \\ 4 & -5 & 7 \end{bmatrix}$.

12 $\begin{bmatrix} -7 \\ 5 \end{bmatrix}$.

14 $\begin{bmatrix} 7 \\ -3 \\ -2 \end{bmatrix}$.

28 Linearly dependent.

Section 4.5

6 $\begin{bmatrix} 3 \\ 6 \\ -9 \\ -3 \end{bmatrix}, \begin{bmatrix} 6 \\ -2 \\ 5 \\ 1 \end{bmatrix}$; dim is 2.

8 $\begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$; dim is 3.

10 2.

14 3, 3.

19 a). T; b). F; c). F; d). F; e). T.

29 a). T; b). T; c). T.

Section 4.6

4 rank $A = 3$; dim Nul $A = 3$; Basis for Col A: $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \\ -3 \\ -2 \end{bmatrix}, \begin{bmatrix} 7 \\ 10 \\ 1 \\ -5 \\ 0 \end{bmatrix}$.

Basis for Row A: $(1, 1, -3, 7, 9, -9), (0, 1, -1, 3, 4, -3), (0, 0, 0, 1, -1, -2)$.

Basis for Nul A: $\begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -9 \\ -7 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -3 \\ 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$.

6 0, 3, 3.

12 2.

14 3, 3.

17 a). T; b). F; c). T; d). F; e). T.

18 a). F; b). F; c). T; d). T; e). T.

24 It is possible that for some $\mathbf{b}$ in $\mathcal{R}^7$, the equation $A\mathbf{x} = \mathbf{b}$ has a unique solution. However, for some right-hans sides, the equation $A\mathbf{x} = \mathbf{b}$ may have no solution.

28 a). Because rank A + dim Nul A = n and dim Row A = rank A, we have dim Row A + dim Nul A = n. b). Because rank A^T + dim Nul A^T = m and dim Col A = dim Row A^T = rank A^T, we have dim Col A + dim Nul A^T = m.

Section 4.7

4 (i)

8 $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P} = \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}$, $\underset{\mathcal{C}\to\mathcal{B}}{P} = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$.

11 a). F; b). T.

Section 5.1

8 Yes. $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$.

12 $4, 0, -3$

20 $\lambda = 0$ and there are two LI eigenvecters: $\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$.

21 a). F; b). T; c). T; d). T; e). F.

24 Proof.

Section 5.2

8 $\lambda^2 - 10\lambda + 25; 5$

12 $-\lambda^3 + 5\lambda^2 - 2\lambda - 8$

20 Proof.

21 a). F; b). F; c). T; d). F.

24 Proof.

Section 5.3

10 $P = \begin{bmatrix} 1 & -3 \\ 1 & 4 \end{bmatrix}$, $D = \begin{bmatrix} 5 & 0 \\ 0 & -2 \end{bmatrix}$.

16 $P = \begin{bmatrix} -2 & -3 & -2 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

22 a). F; b). F; c). T; d). F.

26 Yes.

28 Proof.

Section 5.4

2 $\begin{bmatrix} 2 & -4 \\ -3 & 5 \end{bmatrix}$,

8 $24\mathbf{b}_1 - 20\mathbf{b}_2 + 11\mathbf{b}_3$

12 $\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$.

16 $\mathbf{b}_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$.

20 Proof.