MATH 2164 **Final Examination** **May 2002**

Name:______________________

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1. Find the solution of the system

$$\begin{cases} x_1 + 2x_2 = 5, \\ 2x_1 + x_2 + x_3 = 4. \end{cases}$$

2. Find the solution of the system

$$\begin{cases} x_1 + 2x_2 = 5 \\ 2x_1 + x_2 = 4 \end{cases}$$

by using Cramer's rule.

3. Find the inverse of the matrix $A = \begin{bmatrix} 0 & -2 & -1 \\ 3 & 0 & 0 \\ -1 & 1 & 1 \end{bmatrix}$

(a) by using the row-reduction algorithm.

(b) by computing the adjugate of A first and then obtaining A^{-1}.

4. (a) Combine the methods of row reduction and cofactor expansion to compute the determinant :

$$\begin{vmatrix} 1 & -2 & 5 & 2 \\ 0 & 0 & 3 & 0 \\ 2 & -6 & -7 & 5 \\ 1 & 0 & 4 & 4 \end{vmatrix}.$$

(b) Find the volume of the parallelepiped with one vertex at the origin and adjacent vertices at $(1, 0, -2)$, $(1, 2, 4)$, $(7, 1, 0)$.

5. Let $\mathbf{b}_1 = \begin{bmatrix} 1 \\ 0 \\ -3 \\ 2 \end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \\ -3 \end{bmatrix}$, $\mathbf{b}_3 = \begin{bmatrix} -3 \\ -4 \\ 1 \\ 6 \end{bmatrix}$, $\mathbf{b}_4 = \begin{bmatrix} 1 \\ -3 \\ -8 \\ 7 \end{bmatrix}$, $\mathbf{b}_5 = \begin{bmatrix} 2 \\ 1 \\ -6 \\ 9 \end{bmatrix}$ and $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \cdots, \mathbf{b}_5\}$.

(a) Find a basis for $\mathcal{B}$.

(b) Find dim Col A when $A = [\mathbf{b}_1\ \mathbf{b}_2\ \cdots\ \mathbf{b}_5]$.

6. Find a basis for Nul A and dim Row A, where

$$A = \begin{bmatrix} -2 & 4 & -2 & -4 \\ 2 & -6 & -3 & 1 \\ -3 & 8 & 2 & -3 \end{bmatrix}.$$

7. Define $T : R^2 \to R^2$ by $T(\mathbf{x}) = \begin{bmatrix} 5 & -3 \\ -7 & 1 \end{bmatrix} \mathbf{x}$. Find a basis $\mathcal{B}$ for R^2 with the property that $[T]_{\mathcal{B}}$ is diagonal.

8. Suppose $A = \begin{bmatrix} -1 & 0 & 1 \\ -3 & 4 & 1 \\ 0 & 0 & 2 \end{bmatrix}$.

(a) Show that the characteristic polynomial of A is $-\lambda^3 + 5\lambda^2 - 2\lambda - 8$ $= -(\lambda - 2)(\lambda - 4)(\lambda + 1)$.

(b) Find the eigenvalues of A.

(c) Diagonalize A if possible.

9. (a) Let λ be an eigenvalue of an invertible matrix A. Show that λ^{-1} and λ^{-2} are eigenvalues of A^{-1} and A^{-2} respectively.

(b) Suppose that A is invertible and similar to B. Show that A^{-1} is similar to B^{-1} and A^2 is similar to B^2.

10. Decide whether the following statements are true or false.

(a) A row replacement operation does not affect the determinant of a matrix.

(b) If dim $V = n$ and S is a linear independent set in V, then S is a basis for V.

(c) If A is an $n \times n$ matrix, then the equation $A\mathbf{x} = \mathbf{b}$ has at least one solution for each $\mathbf{b}$ in R^n.

(d) If there exists a linear independent set $\{\mathbf{b}_1, \mathbf{b}_2, \cdots, \mathbf{b}_p\}$ in V, then dim $V \geq p$.

(e) A matrix A is not invertible if and only if 0 is an eigenvalue of A.

(f) If A is diagonalizable, then A has n distinct eigenvalues.