MATH 2164 Final Examination

August 1998

Name : ____________________

1. Find the inverse of the matrix $\begin{bmatrix} 1 & 3 & -1 \\ 0 & 1 & 2 \\ -1 & 0 & 8 \end{bmatrix}$ by using the row-reduction algorithm.

2. Find the solution of the system

$$\begin{cases} x_1 + 2x_2 = 0 \\ 2x_1 + x_2 + x_3 = 2 \\ x_1 + 2x_3 = 0 \end{cases}$$

by using Cramer's rule.

3. **(a.)** Combine the methods of row reduction and cofactor expansion to compute the determinant :

$$\begin{vmatrix} -1 & 2 & 3 & 0 \\ 2 & 1 & 1 & 0 \\ 0 & 3 & 7 & -1 \\ 1 & 0 & -3 & 1 \end{vmatrix}.$$

(b.) Find the area of the parallelogram whose vertices are (0,0), (5,2), (6,4), (11,6).

4. Find bases for Nul A, Col A and Row A, where

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 & 3 \\ 0 & 1 & 2 & 2 & 3 \\ 3 & 2 & 1 & -5 & -3 \\ 1 & 1 & 1 & 4 & 5 \end{bmatrix}.$$

5. Let $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \vec{x} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ and $\mathcal{B} = \{\vec{v}_1, \vec{v}_2\}$.

(a.) Is the set $\mathcal{B}$ a basis for $Span\,\{\vec{v}_1, \vec{v}_2\}$?

(b.) Determine if $\vec{x}$ is in $Span\,\{\vec{v}_1, \vec{v}_2\}$, and if it is, find the coordinate vector of $\vec{x}$ relative to $\mathcal{B}$.

6. Find the value(s) of h for which the vectors $\begin{bmatrix} 1 \\ -5 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ 8 \\ 6 \end{bmatrix}, \begin{bmatrix} 4 \\ h \\ -8 \end{bmatrix}$ are linearly dependent.

7. Suppose $A = \begin{bmatrix} 4 & 0 & -1 \\ 4 & 2 & -2 \\ 2 & 0 & 1 \end{bmatrix}$.

(a.) Show that the characteristic polynomial of A is $(6-5\lambda+\lambda^2)(2-\lambda)$.

(b.) Find the eigenvalues of A.

(c.) Diagonalize A if possible.

8. Suppose $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 5 & 0 & 0 \\ 3 & 6 & 8 & 0 \\ 4 & 7 & 9 & 10 \end{bmatrix}$. Find the eigenvalues of A, A^{-1}, A^T, A^3.

9. Decide whether the following statements are true or false. Give a reason for each answer.

   (a.) Any system of n linear equations in n variables has at most one solution.

   (b.) The columns of an invertible $n \times n$ matrix form a basis for $\mathcal{R}^n$.

(c.) If $AB = C$ and C has 3 columns, then B has 3 columns.

(d.) If A and B are $n \times n$ matrices, then $(A+B)(A-B) = A^2 - B^2$.

(e.) Row operations on a matrix A can change the linear dependence relations among the columns of A.

(f.) Row operations on a matrix A can change the linear dependence relations among the rows of A.

(g.) If $V = \text{Span}\,\{\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_p\}$ and $\dim V = p$, then the set $\{\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_p\}$ cannot be linearly dependent.

(h.) If $p \geq 2$ and $\dim\, V = p$, then every set of $p - 1$ nonzero vectors is linearly independent.

(i.) Suppose that a nonhomogeneous system of four linear equations in seven unknowns has a solution, with three free variables. It is possible to change some constants on the equations' right sides to make the new system inconsistent.

(j.) It is possible for a nonhomogeneous system of five equations in three unknowns to have a unique solution for some right-hand side of constants.

10. Decide whether the following statements are true or false. Give a reason for each answer.

(a.) If A is invertible and 1 is an eigenvalue for A, then 1 is also an eigenvalue for A^{-1}.

(b.) If an $n \times n$ matrix is diagonalizable, then the matrix is invertible.

(c.) A is a 4×4 matrix with three eigenvalues. One eigenspace is one-dimensional, and one of the other eigenspaces is two-dimensional. It is possible that A is not diagonalizable.

(d.) If $AP = PD$, with D diagonal, then the nonzero columns of P must be eigenvectors of A.