MATH 2164 **Test 1** **February 2002**

Name:_____

Show the details of your work !! ID:_____

1. Find the general solutions of the following systems:

(a) $$\begin{cases} x_1 & & - & 5x_3 & = & -14, \\ & 2x_2 & + & 3x_3 & = & 13, \\ 3x_1 & - 4x_2 & + & x_3 & = & -2. \end{cases}$$

(b) $$x_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}.$$

(c) $\begin{bmatrix} 1 & 2 \\ -1 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 5 \end{bmatrix}$.

2. Let $A = \begin{bmatrix} 5 & -3 \\ -15 & 9 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$. Show that the equation $A\vec{x} = \vec{b}$ is not consistent for all possible $\vec{b}$, and describe the set of all $\vec{b}$ for which $A\vec{x} = \vec{b}$ is consistent.

3. (a) Find the solutions of $A\vec{X} = 0$ in parametric vector form, where

$$A = \begin{bmatrix} 1 & -3 & -2 \\ 0 & 1 & -1 \\ -2 & 3 & 7 \end{bmatrix}.$$

(b) Find the solutions of $A\vec{X} = \vec{b}$ in parametric vector form, where A is given in (a), and

$$b = \begin{bmatrix} -5 \\ 4 \\ -2 \end{bmatrix}.$$

Also, give a geometric description of the solution set and compare it with that in (a).

4. Let $A = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 3 & -2 \\ -2 & 6 & 4 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} -4 \\ 1 \\ -4 \end{bmatrix}$. Denote the columns of A by $\vec{a}_1$, $\vec{a}_2$, $\vec{a}_3$ and let $W = \text{Span}\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$.

   (a) Is $\vec{b}$ in W?

   (b) Is $\vec{a}_1$ in W?

5. Determine if the columns of the given matrix form a linearly dependent set:

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & 4 \\ 0 & 3 & 6 \\ 2 & 2 & 6 \end{bmatrix}$$

6. Determine by inspection if the set is linearly independent. Give reasons for your answers.

   (a) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}$;

   (b) $\begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \end{bmatrix}$;

   (c) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ 7 \end{bmatrix}$;

   (d) $\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} -6 \\ 4 \\ -2 \end{bmatrix}$.

7. Suppose T is a linear transformation from R^3 into R^4 and $T(\vec{e}_1) = (-5, 3, -6, 0), T(\vec{e}_2) = (4, -2, 2, 2), T(\vec{e}_3) = (3, -2, 2, 2)$. Find the standard matrix.

8. Determine if the linear transformation T, whose standard matrix is one of the following matrices, is one-to-one and if the linear transformation is onto:

(a) $\begin{bmatrix} 1 & 2 & 3 & 7 \\ 0 & 2 & 3 & 7 \\ 0 & 0 & 3 & 7 \end{bmatrix}$;

(b) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}$;

(c) $\begin{bmatrix} 1 & 2 & 3 & 7 \\ 0 & 2 & 3 & 7 \\ 0 & 0 & 3 & 7 \\ 0 & 0 & 0 & 4 \end{bmatrix}$;

(d) $\begin{bmatrix} 1 & 2 & 3 & 7 \\ 0 & 2 & 3 & 7 \\ 0 & 0 & 3 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.