MATH 2164 Test 1

July 1998

Name : ____________________

1. Find the general solutions of the following systems:

a. $$\left\{\begin{array}{rcrcrcr} 2x_1 & & & - & 4x_3 & = & 8, \\ & & x_2 & + & 3x_3 & = & -2, \\ 3x_1 & + & 5x_2 & + & 8x_3 & = & 3. \end{array}\right.$$

b. $$x_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} + x_3 \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}.$$

c. $\begin{bmatrix} 1 & 2 \\ -1 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -7 \\ 1 \\ 5 \end{bmatrix}.$

2. Determine if $\vec{b}$ is in $Span\,\{\vec{a}_1, \vec{a}_2\}$, where $\vec{b} = \begin{bmatrix} 4 \\ 3 \\ 11 \end{bmatrix}$ and $\vec{a}_1, \vec{a}_2$ are the first and second columns of $A = \begin{bmatrix} 2 & -1 \\ -1 & 3 \\ 1 & 4 \end{bmatrix}$ respectively.

3. Let $\vec{v}_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$. Show that $\begin{bmatrix} h \\ k \end{bmatrix}$ is in $Span\{\vec{v}_1, \vec{v}_2\}$ for all h and k.

4. Find the solutions of $A\vec{X} = 0$ in parametric vector form, where

$$A = \begin{bmatrix} 1 & 2 & -4 & 2 \\ -3 & -5 & 1 & 0 \\ 4 & 7 & -5 & 2 \end{bmatrix}.$$

5. Describe the solution set in R^3 of $x_1 + x_2 - 8x_3 = 0$, and compare it with the solution set of $x_1 + x_2 - 8x_3 = 7$.

6. Determine if the set is linearly independent. Give reasons for your answers.

a. $\begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 7 \end{bmatrix};$

b. $\begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 7 \end{bmatrix}, \begin{bmatrix} 9 \\ 11 \end{bmatrix};$

c. $\begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix};$

d. $\begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} -3 \\ 3 \\ -6 \end{bmatrix}.$

7. Suppose T is a linear transformation from R^2 into R^3 and $T(\vec{e}_1) = (-5, 3, -6), T(\vec{e}_2) = (4, -2, 2)$. Find the standard matrix.

8. Choose h and k such that the system has (a) no solution, (b) a unique solution and (c) many solutions.

$$\left\{ \begin{array}{rcrcl} x_1 & + & 3x_2 & = & 4, \\ 3x_1 & + & hx_2 & = & k. \end{array} \right.$$

9. Determine if the linear transformation T, whose standard matrix is one of the following matrices, is one-to-one and if the linear transformation is onto:

a. $\begin{bmatrix} 1 & 2 & 4 & 6 \\ 0 & 1 & 5 & 7 \\ 0 & 0 & 1 & 8 \end{bmatrix}$;

b. $\begin{bmatrix} 1 & 2 & 4 & 6 \\ 0 & 1 & 5 & 7 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$;

c. $\begin{bmatrix} 1 & 2 & 4 & 6 \\ 0 & 1 & 5 & 7 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 2 \end{bmatrix}$;

d. $\begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.