MATH 2164 **Test 2** **March 2002**

Name:________________________

Show the details of your work !! ID:________________________

1. Let

$$A = \begin{bmatrix} -1 & 2 & 3 \\ 4 & 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix}$$

Find the indicated matrices, if it is defined; if it is undefined, explain why.

(a) 3A+B

(b) AB

(c) CA

2. Determine whether or not the matrix

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 3 & 0 & 4 \\ 5 & 0 & 7 & 0 \\ 4 & 0 & 2 & 4 \end{bmatrix}$$

is invertible in two different ways:

(a) Using the row reduction method

(b) Calculating the determinant by using the cofactor expansion method

3. Find the inverse of the matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

(a) by using the row reduction method

(b) by the formula

$$\mathbf{A}^{-1} = \frac{1}{\det\mathbf{A}}\mathrm{adj}\mathbf{A}$$

4. Suppose that A, B, C, X, Y, Z are $n \times n$ matrices and A, C are invertible. Find X, Y and Z so that

$$\begin{bmatrix} X & 0 \\ Y & Z \end{bmatrix} \begin{bmatrix} A & 0 \\ B & C \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

5. Find an LU factorization of the matrix

$$\begin{bmatrix} 3 & -6 & 3 \\ 6 & -7 & 2 \\ -3 & 21 & 0 \end{bmatrix}$$

6. Use Cramer's rule to compute the solution of the system

$$\begin{cases} x_1 + 3x_2 &= 7, \\ 2x_1 + 4x_2 &= 10. \end{cases}$$

7. Combine the methods of row reduction and cofactor expansion to compute the determinant :

$$\begin{vmatrix} -1 & 2 & 3 & 0 \\ 1 & 4 & 3 & 0 \\ 8 & 4 & 7 & 6 \\ 4 & 2 & 4 & 3 \end{vmatrix}.$$

8. Suppose that an economy is divided into two sectors — the sector A and the sector B. For each unit of output, the sector A requires 0.10 unit from other companies in the sector, 0.50 unit from the sector B. For each unit of output, the sector B uses 0.20 unit of its own output, 0.60 unit from the sector A. Construct the consumption matrix for this economy and determine the production levels needed to satisfy a final demand of 18 units for the sector A and 11 units for the sector B.

9. Find the area of the parallelgram whose vertices are (0,0), (5,2), (6.4) and (11,6).

10. Decide whether the following statements are true or false.

(a) If A is invertible, then the inverse of A^{-1} is A itself.

(b) If A is an $n \times n$ matrix, then the equation $A\mathbf{x} = \mathbf{b}$ has at least one solution for each $\mathbf{b}$ in R^n.

(c) $(A + B)^{-1} = A^{-1}B^{-1}$.

(d) If A and P are square matrices, with P invertible, then $\det PAP^{-1} = \det A$.

(e) If A is a 2×2 matrix and $\det A = 5$, then $\det(3A) = 15$.

(f) Suppose that A is an $n \times n$ matrix. If the equation $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution, then A has fewer than n pivot positions.