MATH 2164 Test 2

August 1998

Name : ______________________

1. Suppose $A = \begin{bmatrix} -1 & 2 \\ 4 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix}$. Find $2AB$ and $2BA$.

2. Use Cramer's rule to compute the solution of the system

$$\begin{cases} 2x_1 + 4x_2 & = & 6, \\ 3x_1 + 5x_2 & = & 7. \end{cases}$$

3. **(a.)** Combine the methods of row reduction and cofactor expansion to compute the determinant :

$$\begin{vmatrix} -1 & 2 & 3 & 0 \\ 3 & 4 & 3 & 0 \\ 8 & 4 & 6 & 6 \\ 4 & 2 & 4 & 3 \end{vmatrix}.$$

(b.) Suppose that A and B are 2×2 matrices, det $A = 5$ and det B $= 3$, Find out det A^T, det AB, det BA and det $2A$.

4. Determine which of the following matrices are invertible by using the row reduction method :

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 3 & 0 & 4 \\ 5 & 6 & 7 & 8 \\ 4 & 3 & 5 & 4 \end{bmatrix}.$$

5. Compute the adjugate of the given matrix and use this result to give the inverse of the matrix

$$A = \begin{bmatrix} 3 & 5 & 4 \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix}.$$

6. Find bases for Nul A, Col A and Row A, where

$$A = \begin{bmatrix} 1 & 1 & 4 & 1 & 5 \\ 0 & 1 & 2 & 2 & 6 \\ 3 & 1 & 8 & -1 & 3 \\ 1 & 1 & 4 & 4 & 8 \end{bmatrix}.$$

7. Let $\vec{v}_1 = \begin{bmatrix} 2 \\ 6 \\ 2 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \vec{x} = \begin{bmatrix} 5 \\ 18 \\ 7 \end{bmatrix}$ and $\mathcal{B} = \{\vec{v}_1, \vec{v}_2\}$.

(a.) Is the set $\mathcal{B}$ a basis for $Span\,\{\vec{v}_1, \vec{v}_2\}$?

(b.) Determine if $\vec{x}$ is in $Span\,\{\vec{v}_1, \vec{v}_2\}$, and if it is, find the coordinate vector of $\vec{x}$ relative to $\mathcal{B}$.

8. **(a.)** Let A be a 7×4 matrix. Suppose rank $A = 3$, find dim Nul A, dim Col A, dim Row A and rank A^T.

(b.) Let A be a 4×7 matrix. Suppose dim Nul $A = 3$, find rank A, dim Col A, dim Row A and rank A^T.

(c.) If A is a 7×5 matrix, what is the largest possible rank of A ? If A is a 5×7 matrix, what is the largest possible rank of A ? Explain your answers.

(d.) Suppose A is a 6×8 matrix, what is the smallest possible dimension of Nul A ? Explain your answer.

9. Decide whether the following statements are true or false. Give a reason for each answer.

(a.) It is possible for a 5×5 matrix A to be invertible when its columns do not span $\mathcal{R}^5$.

(b.) If A is invertible, then the columns of A^{-1} are linearly independent.

(c.) If A and B are $m \times n$ matrices, then both AB^T and BA^T are defined.

(d.) If A and B are $n \times n$ matrices, then $(A+B)(A-B) = A^2 - B^2$.

(e.) Row operations on a matrix A can change the linear dependence relations among the rows of A.

(f.) Row operations on a matrix A can change the linear dependence relations among the columns of A.

(g.) If there exists a linearly dependent set $\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_p$ in V, then $\dim V \leq p$.

(h.) If there exists a linearly independent set $\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_p$ in V, then $\dim V \geq p$.