MATH 2171 Final Examination December 2001

Name : ____________________

SHOW THE DETAILS OF YOUR WORK ID : ____________________

1. Find the general solution of the following problem :

$$\frac{dy}{dx} = (1+y)(e^x + e^{2x}).$$

2. Solve the initial value problem :

$$\frac{dy}{dx} - 2y = 3e^{2x}, \quad y(0) = 4.$$

3. (a) Find the general solution of the following problem :

$$y'' + 8y' + 25y = 0.$$

(b) Solve the initial value problem :

$$y'' - 4y' + 3y = 0, \quad y(0) = 3, \quad y'(0) = 5.$$

4. Find the general solution to the following equation using the method of undetermined coefficients :

$$y'' - 3y' + 2y = 20 \sin 2t + e^t.$$

5. Find the Laplace transform $\mathcal{L}(f)$ of the following two functions:

(a) $f(t) = 2 + 3e^{4t} + e^{-t}\cos 2t$;

(b) $f(t) = \sin(t-2)u(t-2) + \begin{cases} t, & 0 \le t < 1 \\ 0, & 1 \le t. \end{cases}$

Find $f(t)$ if $F(s) = \mathcal{L}(f)$ equals

(c) $F(s) = \dfrac{s}{s^2+4} + \dfrac{s}{s^2+4s+13}$;

(d) $F(s) = \dfrac{s - se^{-2s}}{s^2 - 6s + 8}$.

6. Solve the initial-value problem

$$y'' + 3y' + 2y = 6\delta(t-3), \quad y(0) = 0, \quad y'(0) = 1.$$

7. Solve the given system subject to the indicated initial condition

$$\mathbf{x}' = \begin{pmatrix} 1 & -6 \\ 1 & -4 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 7 \\ 3 \end{pmatrix}.$$

8. Find the general solution of the problem:

$$\mathbf{x}' = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} \mathbf{x}.$$

9. Let

$$A = \begin{pmatrix} 4 & 1 & 4 \\ 1 & 7 & 1 \\ 4 & 1 & 4 \end{pmatrix}.$$

(a) Show

$$\det(A - \lambda I) = -\lambda^3 + 15\lambda^2 - 54\lambda = -\lambda(\lambda - 6)(\lambda - 9).$$

(b) Find the general solution of the given system :

$$\mathbf{x}' = A\mathbf{x}.$$

10. Suppose that the motion of a mass-and-spring system is described by the following system:

$$\mathbf{x}'' = \begin{pmatrix} -5 & 4 \\ 4 & -5 \end{pmatrix} \mathbf{x}.$$

Find the two natural frequencies of the system and describe its two natural models of oscillation.