MATH 2171 **Final Examination** **May 2001**

Name : ____________________

SHOW THE DETAILS OF YOUR WORK ID : ____________________

1. (a) Find the general solution of the following problem :

$$\frac{dy}{dt} + \frac{2}{t}y = t^3.$$

(b) Solve the initial value problem :

$$\frac{dy}{dt} - 3y = 2e^{4t}, \quad y(0) = 1.$$

2. (a) Find the general solution of the following problem :

$$y'' + 6y' + 13y = 0.$$

(b) Solve the initial value problem :

$$y'' - 5y' + 4y = 0, \quad y(0) = 3, \quad y'(0) = 0.$$

3. Find the general solution to the following equation using the method of undetermined coefficients :

$$y'' - 2y' + y = 25 \sin 2t + t.$$

4. Solve the given system subject to the indicated initial condition

$$\frac{d\vec{x}}{dt} = \begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix} \vec{x}, \quad \vec{x}(0) = \begin{pmatrix} 3 \\ 2 \end{pmatrix}.$$

5. Let

$$A = \begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix}.$$

(a) Show

$$\det(A - \lambda I) = (1 - \lambda)(\lambda - 3)(\lambda + 2).$$

(b) Find the general solution of the given system :

$$\frac{d\vec{x}}{dt} = A\vec{x}.$$

6. Find the Laplace transform $\mathcal{L}(f)$ of the following two functions:

(a) $f(t) = 3 + 4e^{5t} + e^{-t}\cos 2t;$

(b) $f(t) = \sin(t-2)u_2(t) + tu_3(t)$.

7. Find $f(t)$ if $F(s) = \mathcal{L}(f)$ equals

(a) $F(s) = \dfrac{1}{s^2+s-2} + \dfrac{6}{s^2+6s+13}$;

(b) $F(s) = \dfrac{2(s-2)e^{-2s}}{s^2-4s+3}$.

8. Solve the initial-value problem

$$y' + 2y = u_2(t), \quad y(0) = 1.$$

9. Solve the initial-value problem

$$y'' + 9y = 6\delta(t-2), \quad y(0) = 1, \quad y'(0) = 0.$$