MATH 2171 **Test 1** **Spring 2001**

Name : _____

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1. Solve the following initial-value problem

$$y' + 3y = e^{-t}, \quad y(0) = 0.$$

2. Find approximate values of the solution of the given problem

$$y' = 0.5 - t^2 + y^2, \quad y(0) = 2$$

at $t = 0.1$ and 0.2 using the Euler method with $h = 0.1$.

3. For the problem : $\dfrac{dy}{dt} = y(y-3)(y-5),\ y_0 \geq 0$, find all the equilibrium solutions and classify each one as asymptotically stable or unstable.

4. Radium-226 has a half-life of 1620 years. Find the time period during which a given mass of this material is reduced by one-quarter.

5. Suppose that a certain population obeys the logistic equation

$$\frac{dy}{dt} = ry[1 - (y/K)].$$

If $y_0 = K/4$, find the time τ at which the initial population has doubled. Find the value of τ corresponding to $r = 0.05$ per year. (Hint : For the initial value problem

$$\frac{dy}{dt} = ry(1 - y/K), \quad y(0) = y_0,$$

the solution is

$$y(t) = \frac{y_0 K}{y_0 + (K - y_0)e^{-rt}}.)$$

6. Find the general solution of the following differential equation:

$$\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}.$$

7. Find the general solution of the following differential equation:

$$y' = 2x(y^2 - 9).$$