MATH 2171 **Test III** **Spring 2001**

Name : _____

SHOW THE DETAILS OF YOUR WORK ID : _____

1. a. Transform the given equation into a system of first order equations:

$$u'' + 2u' + 3u = 0;$$

b. Transform the given system into a single equation of second order about the first component of $\vec{x}$:

$$\frac{d\vec{x}}{dt} = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} \vec{x}.$$

2. Solve the initial value problem

$$\frac{d\vec{x}}{dt} = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} \vec{x}, \quad \vec{x}(0) = \begin{pmatrix} 3 \\ 1 \end{pmatrix}.$$

3. Solve the initial value problem

$$\frac{d\vec{x}}{dt} = \begin{pmatrix} -6 & 5 \\ -5 & 4 \end{pmatrix} \vec{x}, \quad \vec{x}(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

4. Find the general solution of the given system

$$\frac{d\vec{x}}{dt} = \begin{pmatrix} 1 & 2 \\ -5 & -1 \end{pmatrix} \vec{x}.$$

5. Find the general solution of the given system

$$\frac{d\vec{x}}{dt} = \begin{pmatrix} 0 & 0 & -1 \\ 2 & 0 & 0 \\ -1 & 2 & 4 \end{pmatrix} \vec{x}.$$

(Hint : $-\lambda^3 + 4\lambda^2 + \lambda - 4 = (\lambda^2 - 1)(4 - \lambda)$.)