MATH 6171 Answers for Homework 3 Fall 2005

For even-numbered problems in Chapters 10 & 11 which you have turned in only

Section 10.1

2 $\frac{2\pi}{n}, \frac{2\pi}{n}, k, k, \frac{k}{n}, \frac{k}{n}$.

6 (It is a proof.)

Section 10.2

8 $\frac{4\pi^2}{3} + 4\left(\cos x + \frac{1}{4}\cos 2x + \frac{1}{9}\cos 3x + \cdots\right) - 4\pi\left(\sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + \cdots\right)$

10 $\frac{\pi}{2} - \frac{4}{\pi}\left(\cos x + \frac{1}{9}\cos 3x + \frac{1}{25}\cos 5x + \cdots\right) + 2\left(\sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x - \frac{1}{4}\sin 4x + - \cdots\right)$

Section 10.3

4 $1 - \frac{8}{\pi^2}\left(\cos\frac{\pi x}{2} + \frac{1}{9}\cos\frac{3\pi x}{2} + \frac{1}{25}\cos\frac{5\pi x}{2} + \cdots\right)$

8 $\frac{1}{4} + \frac{2}{\pi^2}\left(\cos 2\pi x + \frac{1}{9}\cos 6\pi x + \frac{1}{25}\cos 10\pi x + \cdots\right)$

Section 10.4

2 Even: $|x|, e^{x^2}, \sin^2 x, x\sin x, e^{-|x|}$. Odd: $x\cos x$.

8 Neither even nor odd.

10a (It is a proof.)

18 $\frac{\pi^2}{2} = \frac{\pi^2}{6} + 2\left(1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots\right)$

Section 10.8

2 (It is a proof.)

Section 10.10

2 $\frac{1}{(k+iw)\sqrt{2\pi}}$.

16 (They are proofs and derivations.)

Section 11.1

2 (Verification)

4 (Verification)

6 (Verification)

10 (Verification)

22 $u = c(x)e^y + h(x)$

Section 11.3

4 $\frac{0.8}{\pi}\left(\cos t \sin x + \frac{1}{3^3}\cos 3t \sin 3x + \frac{1}{5^3}\cos 5t \sin 5x + \cdots\right)$

8 $\frac{4}{5\pi}\left(\frac{1}{4}\cos 2t \sin 2x - \frac{1}{36}\cos 6t \sin 6x + \frac{1}{100}\cos 10t \sin 10x - + \cdots\right)$

14 $u = ce^{k(x^3+y^3)}$

16 $u = \left(c_1 e^{kx} + c_2 e^{-kx}\right)(A\cos ky + B\sin ky)$, or $u = \left(c_1 e^{ky} + c_2 e^{-ky}\right)(A\cos kx + B\sin kx)$, or $u = (ax+b)(cy+d)$

Section 11.4

8 (Sketch a figure.)

12 (Verification)

Section 11.5

2 $\lambda_1^2 = \frac{\ln 2}{20}, c^2 = \frac{L^2 \ln 2}{20\pi^2} = 0.0035L^2.$

14 $w(x) = \frac{N}{c^2\alpha^2}\left[-e^{-\alpha x} - \frac{1}{L}\left(1 - e^{-\alpha L}\right)x + 1\right]$

Section 11.6

2 $A(p) = e^{-p}, B(p) = 0$, and $u(x,t) = \int_0^\infty e^{-(p+c^2P^2t)}\cos pxdp$

10e $u(x,t) = \frac{U_0}{2}\left[\text{erf}\frac{1-x}{2c\sqrt{t}} + \text{erf}\frac{1+x}{2c\sqrt{t}}\right], \quad (t > 0)$

Section 11.8

8 $B_{mn} = \frac{(-1)^{m+n}4}{mn}$

Section 11.11

4 $u = -\frac{80 \ln r}{\ln 2} + 300$

6 $u = \frac{c}{r} + k$

12 $u = r^2 \left(-\cos^2 \phi + \frac{1}{3}\right) + \frac{2}{3}$

14 $u = 4r^3 P_3(\cos\phi) - 2r^2 P_2(\cos\phi) + r P_1(\cos\phi) - 2$

16 $u_{int} = \frac{4}{3} r^2 P_2(\cos\phi) - \frac{1}{3}, \quad u_{ext} = \frac{4}{3r^3} P_2(\cos\phi) - \frac{1}{3r}$

Section 11.12

4 $u(x,t) = t + 1 - (t - x^2)u(t - x^2)$