MATH 6171 **Test 1** **Fall 2001**

Name : _____
ID: _____

Show the details of your work !!

1. For each of the following equations, determine whether the power series method works near $x = 0$ and whether the Frobenius method works (Justify your answer).

   (a) $x^2y'' - (x+6)y' - 3y = 0$;

   (b) $(x+1)y'' - (x+2)y' - (x+3)y = 0$;

   (c) $x^2y'' - xy' - (x^2+3)y = 0$.

2. Find one solution of the following equation using the Frobenius method (suppose $x_0 = 0$) :

$$xy'' + 2y' + 4xy = 0.$$

3. Show orthogonality on the given interval and determine the corresponding orthonormal set of functions

$$1,\ \cos 8nx,\ \sin 8nx, \quad n = 1, 2, \cdots, \quad (0 \leq x \leq \pi/4).$$

4. Find the Laplace transform $\mathcal{L}\{f\}$ of the given functions:

   (a) $f(t) = 2t + e^{3t}\cos 4t + e^{5t}\sinh 6t$;

   (b) $f(t) = \sin(2t+3) + e^{2+3t}$;

   (c) $f(t) = t^2 u(t-3) + \delta(t-6)$.

5. Find $f(t)$ if $F(s) = \mathcal{L}\{f\}$ equals

(a) $F(s) = \dfrac{1}{s^2 + 4s + 13} + \dfrac{1}{s^2}$;

(b) $F(s) = \dfrac{e^{-s}}{s^2 - 9}$;

(c) $F(s) = \dfrac{1}{s^2(s-1)}$.

6. Solve the given initial value problem

$$y_1' = 5y_1 + y_2, \;\; y_2' = y_1 + 5y_2, \;\; y_1(0) = -3, \;\; y_2(0) = 7.$$