MATH 6171 **Test 1** **Fall 2002**

Name : _____

Show the details of your work !! ID: _____

1. For each of the following equations, determine whether the power series method works near $x = 0$ and whether the Frobenius method works (Justify your answer).

   (a) $x^3y'' + xy' + (x^2 - 3)y = 0$.

   (b) $x^2y'' + xy' + (x^2 - 3)y = 0$;

   (c) $y'' + xy' + (x^2 - 3)y = 0$;

2. Find one solution of the following equation using the Frobenius method (suppose $x_0 = 0$):

$$x(1-x)y'' + 2(1-2x)y' - 2y = 0.$$

3. Find the eigenvalus and eigenfunctions of the following problem:

$$y'' + \lambda y = 0, \quad y'(0) = y'(1) = 0.$$

4. Find the Laplace transform $\mathcal{L}\{f\}$ of the given functions:

(a) $f(t) = t^2 + e^{2t}\sin 3t + \delta(t-4)$;

(b) $f(t) = \sin^2 t$;

(c) $f(t) = \begin{cases} t, & \text{if } 0 < t < 1, \\ 0, & \text{if } 1 < t. \end{cases}$

(d) $f(t) = u(t-3)\cos t$.

5. Find $f(t)$ if $F(s) = \mathcal{L}\{f\}$ equals

(a) $F(s) = \dfrac{s+4}{s^2+4}$;

(b) $F(s) = \dfrac{1}{(s-2)(s+1)}$;

(c) $F(s) = \dfrac{1}{s^2+2s+10}$;

(d) $F(s) = \dfrac{se^{-2s}}{s^2+9}$;

6. Solve the given initial value problem

$$y'' - y' - 12y = u(t-5), \;\; y(0) = 0, \;\; y'(0) = 1.$$