MATH 6171 **Test 2** **Fall 2001**

Name : ____________________

Show the details of your work !! ID: ____________________

1. (a) Find the area of the triangle with vertices $(1, 2, 3)$, $(1, 2, 4)$, $(2, 2, 3)$.

   (b) Find the volume of the parallelepiped with edge vectors $(1, 0, -7)$, $(0, 1, 1)$, $(2, 1, 0)$.

   (c) Find the length of the curve defined by $\vec{r}(t) = t\vec{i} + \frac{4}{3}t^{\frac{3}{2}}\vec{j}$, where $0 \leq t \leq 6$.

2. (a) div $(xz\vec{i} + (y-z)^2\vec{j} + xyz\vec{k}) =$

(b) curl $(xy^2z^3\vec{i} + \sin x\vec{j}) =$

(c) Find the directional derivative of $f = 3x^2 + 2y^2 + z^2$ at the point $(1, 2, 3)$ in the direction $\vec{a} = \vec{i} + \vec{j} + \vec{k}$.

3. Suppose that f is a smooth function and $\vec{u}$, $\vec{v}$ are smooth vector functions. Show

   (a) div (curl $\vec{u}$) $= 0$;

   (b) curl $(f\vec{v}) =$ (grad f)$\times\vec{v}+f$ curl $\vec{v}$.

4. Evaluate the surface integral $\int\int_S (y^2\vec{i}+x^2\vec{j}+z^2\vec{k})\cdot\vec{n}dA$ by the divergence theorem of Gauss, where S is the surface of the box: $0 \leq x \leq 1, 0 \leq y \leq 4, 0 \leq z \leq 2$.

5. Show that $\vec{F} = (3x^2 + ye^{xy})\vec{i} + (xe^{xy} - z\sin(yz))\vec{j} - (y\sin(yz) + e^{2z})\vec{k}$ has a potential and find a potential function of $\vec{F}$ and evaluate $\int_{(0,0,1)}^{(1,2,3)} \vec{F} \cdot d\vec{r}$ using the potential function.

6. Evaluate the surface integral $\int\int_S (x\vec{i} + y\vec{j} + 3z^2\vec{k}) \cdot \vec{n} dA$, where S is the surface: $z = x^2 + y^2,\ z \leq 4$.

7. Evaluate $\oint_C [(y+2)\vec{i} + (3x+4)\vec{j} + 5z\vec{k}] \cdot d\vec{r}$ by Stokes' theorem, where C is the boundary of the triangle with vertices $(2,0,0)$, $(0,2,0)$ and $(0,0,2)$. (The direction is clockwise as seen by a person standing at the origin.)