MATH 6171 **Test 2** **Fall 2002**

For credits, show all work !!

Name: ______________________

ID: ________________________

1. (a) grad (div $(x^2z\mathbf{i} + (y-z)^2\mathbf{j} + xy\mathbf{k})) =$

   (b) curl $(\cos(yz)\mathbf{i} + \sin x\mathbf{j}) =$

   (c) Find a unit normal vector of the surface $z = \sqrt{x^2 + y^2}$ at the point $\mathbf{P}$: $(3, 4, 5)$.

2. (a) Find the volume of a parallelepiped if the edge vectors are $(1, 2, 3)$, $(1, 2, 4)$, $(2, 2, 5)$.

(b) Find an equation of the plane through $(1, 2, 3)$, $(0, 1, 1)$, $(2, 2, 0)$.

(c) Represent the following curve parametrically: $4x^2 + (y - 1)^2 = 9, \ \ y = z$.

(d) Find the length of the curve defined by $\mathbf{r}(t) = t\mathbf{i} + t^{3/2}\mathbf{j}$, where $0 \le t \le 4$.

3. Assuming sufficient differentiability of the function f and the vector functions $\mathbf{u}$ and $\mathbf{v}$. Show

   (a) curl (grad f) $= 0$;

   (b) curl $(\mathbf{u} + \mathbf{v})$=curl $\mathbf{u}$+curl $\mathbf{v}$.

4. Calculate $\int_C \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r}$ for the following data: $\mathbf{F} = [2x, y, -z], \quad C: \mathbf{r} = [2t, \cos t, \sin t]$ from $(0, 1, 0)$ to $(4\pi, 1, 0)$.

5. Evaluate the surface integral $\int\int_S (x\mathbf{i} + y\mathbf{j} + 3z^2\mathbf{k}) \cdot \mathbf{n} dA$, where S is the surface: $z = x^2 + y^2,\ z \leq 4$.

6. Show that $\mathbf{F} = (x + ye^{xy})\mathbf{i} + (xe^{xy} + z\cos(yz))\mathbf{j} + (y\cos(yz) + z^2)\mathbf{k}$ has a potential and find a potential function of $\mathbf{F}$ and evaluate $\int_{(0,0,0)}^{(1,1,1)} \mathbf{F} \cdot d\mathbf{r}$ using the potential function.

7. Evaluate $\oint_C [(2z + e^{x^3})\mathbf{i} - x\mathbf{j} + x\mathbf{k}] \cdot d\mathbf{r}$ by Stokes' theorem, where C is the $x^2 + y^2 = 1, \;\; z = y + 1$. (clockwise as seen by a person standing at the origin.)