MATH 6171 **Test III** **Fall 2001**

Name :____________________

ID :____________________

Show the details of your work !!

1. Are the following functions even, odd or neither odd nor even?

   (a) $x \sin^2 x$;

   (b) $|f(x)|$, where $f(x)$ is any odd function defined for all x;

   (c) $|f(x)|$, where $f(x)$ is any function defined for all x;

   (d) $g(x) + g(-x)$, $g(x)$ is any function defined for all x.

2. (a) Represent the following periodic function of period $p = 2\pi$ by the Fourier series:

$$f(x) = |x|, \quad -\pi \leq x \leq \pi.$$

(b) Using the Fourier series you get, find the value of the following series

$$1 + \frac{1}{9} + \frac{1}{25} + \cdots + \frac{1}{(2n+1)^2} + \cdots.$$

3. Laplace's equation in spherical coordinates is

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \phi^2} + \frac{cot\phi}{r^2}\frac{\partial u}{\partial \phi} + \frac{1}{r^2 \sin^2\phi}\frac{\partial^2 u}{\partial \theta^2} = 0.$$

u is a solution of this equation and depends on r only. Show that u must be in the form

$$u = \frac{c}{r} + k,$$

where c and k are constants.

4. (a) For $f(t) = t^3$, solve the following problem by the Laplace transform

$$\frac{\partial u}{\partial x} + x^2 \frac{\partial u}{\partial t} = 0, \quad u(x,0) = 0, \quad u(0,t) = f(t), \quad x \geq 0, \quad t \geq 0.$$

(b) For $f(t) = e^{\cos t}$, solve the problem given in (a).

5. (a) Find the solutions of the equation $x(y')^2 + 2yy' = 0$ and show that the solutions can be written as $y = \textit{constant}$ and $x^2 y = \textit{constant}$.

   (b) Let $v = y$, $z = x^2 y$ and $U(v, z) = U(v(x, y), z(x, y)) \equiv u(x, y)$. Suppose that $u(x, y)$ satisfies $x\dfrac{\partial^2 u}{\partial x^2} - 2y\dfrac{\partial^2 u}{\partial x \partial y} + \dfrac{\partial u}{\partial x} = 0$. Show that $U(v, z)$ satisfies $\dfrac{\partial^2 U}{\partial v \partial z} = 0$.

   (c) Find the general solution of $\dfrac{\partial^2 U}{\partial v \partial z} = 0$ and based on the results given in the above part, find the general solution of $x\dfrac{\partial^2 u}{\partial x^2} - 2y\dfrac{\partial^2 u}{\partial x \partial y} + \dfrac{\partial u}{\partial x} = 0$.

6. Solve the problem

$$\begin{cases} \dfrac{\partial u}{\partial t} = c^2 \dfrac{\partial^2 u}{\partial x^2}, & 0 \le x \le L, \quad t \ge 0, \\ u_x(0,t) = u_x(L,t) = 0, & t \ge 0, \\ u(x,0) = f(x), & 0 \le x \le L \end{cases}$$

by the method of separating variables.