MATH 6171 **Test III** **Fall 2002**

Name :____________________

Show the details of your work !! ID :____________________

1. **(a)** Is the function $|x^3|$ odd, even or neither odd nor even?

   (b) Is the function $\sin x^3 + x \cos x$ odd, even or neither odd nor even?

   (c) Is the function $xe^x + x \sin x$ odd, even or neither odd nor even?

   (d) What is the smallest positive period of the function $\sin 2x + \cos 4x$?

   (e) What is the smallest positive period of the function $\cos x + \cos \frac{x}{2} + \cos 2x$?

2. **(a)** Representing the following function by the Fourier cosine series:

$$f(x) = \sin x, \quad 0 < x < \pi.$$

(b) Using the Fourier cosine series you get, show that

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \cdots = \frac{1}{2}.$$

3. Let r, θ, ϕ be the spherical coordinates used in the text. Find the potential in the interior of sphere $R = 1$, assuming that there are no charges in the interior and the potential on the surface is:

$$f(\phi) = 1 - \cos\phi - 6\cos^2\phi.$$

(Hint: The Legendre polynomials of degrees 0, 1, 2 are

$$P_0(x) = 1, \quad P_1(x) = x, \quad P_2(x) = \frac{1}{2}(3x^2 - 1)$$

and

$$\int_{-1}^{1} P_m^2(x)dx = \frac{2}{2m+1},$$

where $P_m(x)$ is the Legendre polynomial of degree m.)

4. (a) Find the solutions of the equation $(y')^2 + 5y' + 4 = 0$ and show that the solutions can be written as $x+y = constant$ and $4x+y = constant$.

(b) Let $v = x + y$, $z = 4x + y$ and $U(v, z) = U(v(x, y), z(x, y)) \equiv u(x, y)$. Suppose that $u(x, y)$ satisfies $\frac{\partial^2 u}{\partial x^2} - 5\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial y^2} = 0$. Show that $U(v, z)$ satisfies $\frac{\partial^2 U}{\partial v \partial z} = 0$.

(c) Find the general solution of $\frac{\partial^2 U}{\partial v \partial z} = 0$ and based on the results given in the above part, find the general solution of $\frac{\partial^2 u}{\partial x^2} - 5\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial y^2} = 0$.

5. **(a)** For $f(t) = t$, solve the following problem by the Laplace transform

$$\frac{\partial v}{\partial x} + x^3 \frac{\partial v}{\partial t} = 6x^3, \quad v(x,0) = 0, \quad v(0,t) = f(t), \quad x \geq 0, \quad t \geq 0.$$

(b) For $f(t) = e^{\sin 2t}$, solve the problem given in (a).

6. Solve the problem

$$\begin{cases} \dfrac{\partial u}{\partial t} = c^2 \dfrac{\partial^2 u}{\partial x^2}, & 0 \le x \le L, \quad t \ge 0, \\ u(0,t) = u_x(L,t) = 0, & t \ge 0, \\ u(x,0) = f(x), & 0 \le x \le L \end{cases}$$

by the method of separating variables.