MATH 6172 **Test I** **Spring 2002**

Name : _____

SHOW THE DETAILS OF YOUR WORK ID : _____

1. (a) Find Re $\frac{z}{\bar{z}}$

(b) Find Im $\frac{1}{z}$

(c) Find the polar form of $\frac{1-i}{1+i}$

(d) Find Arg(1+i)

(e) Find all roots of $\sqrt[3]{216}$

2. Are the following functions harmonic? If so, find a corresponding conjugate function $u(x, y)$ for each of them so that $f(z) = u(x, y) + iv(x, y)$ is an analytic function.

   (a) $v = x^2 - y^2$

   (b) $v = \sin x \cosh y$

3. (a) Find and sketch the image of the region: $\ln 3 < x < \ln 5$ under the mapping $w = e^{2z}$.

(b) Find and sketch the image of the region: $1 < y < 2$ under the mapping $w = \sin z$.

4. (a) Find the linear fractional transformation that maps $-1, 0, 1$ onto $-1, 0, 3$ respectively.

(b) Find a linear fractional transformation that maps $|z| \leq 1$ onto $|w| \leq 1$ such that $z = 1/3$ is mapped onto $w = 0$.

(c) Show that substituting any linear fractional transformation into a linear fractional transformation gives another linear fractional transformation.

5. Integrate

(a) $\int_C \cos^2 z dz$, C is the path from i to 1 along the unit circle.

(b) $\int_C z^2 e^{z^3} dz$, C is the path from 2 along the axes to i.

(c) Show that $\bar{z} = x - iy$ is not analytic and calculate $\int_C \bar{z} dz$, C is the unit circle, counterclockwise.

6. Integrate the following $f(z)$ around the contour C in the counterclockwise sense.

   (a) $f(z) = \dfrac{z^3}{3z-1} + \dfrac{1+z}{z-3}, \quad C : |z| = 1$

   (b) $f(z) = \dfrac{(z+2)^4 + \cos 2z}{(z+1)^2}, \quad C$ is the boundary of square with vertices $\pm\frac{3}{2},\ \pm\frac{3}{2}i$

   (c) $f(z) = \dfrac{\ln(z-2)}{(2z-1)^3}, \quad C : |z-i| = 2$