Math 6172 Answers to Problems in Test I Spring 2002

1. (a)

$$\mathrm{Re}\frac{z}{\bar{z}} = \frac{x^2-y^2}{x^2+y^2}.$$

(b)

$$\mathrm{Im}\frac{1}{z} = -\frac{y}{x^2+y^2}.$$

(c)

$$\frac{1-i}{1+i} = e^{-\frac{\pi}{2}i}.$$

(d)

$$Arg\{1+i\} = \frac{\pi}{4}.$$

(e)

$$6, \quad 6e^{\frac{2\pi}{3}i}, \quad 6e^{\frac{4\pi}{3}i}.$$

2. (a) Yes.

$$u = -2yx+c, \quad f(z) = iz^2+c.$$

(b) Yes.

$$u = -\cos x \sinh y + c, \quad f(z) = i\sin z + c.$$

3. (a) The image on $w$-plane ($w = u+vi$) is

$$3^2 \leq R \leq 5^2,$$

where $R = \sqrt{u^2+v^2}$.

(b) The image on $w$-plane ($w = u+vi$) is the region between the two curves:

$$\frac{u^2}{\cosh^2 1} + \frac{v^2}{\sinh^2 1} = 1$$

and

$$\frac{u^2}{\cosh^2 2} + \frac{v^2}{\sinh^2 2} = 1.$$

4. (a)

$$w = \frac{-3z}{z-2}.$$

(b)

$$w = \frac{3z-1}{z-3}.$$

(c)

$$w_1 = \frac{a_1 z + b_1}{c_1 z + d_1},$$

$$w = \frac{aw_1+b}{cw_1+d} = \frac{a\frac{a_1z+b_1}{c_1z+d_1}+b}{c\frac{a_1z+b_1}{c_1z+d_1}+d} = \frac{aa_1z+ab_1+bc_1z+bd_1}{ca_1z+cb_1+dc_1z+d_1d}$$

$$= \frac{(aa_1+bc_1)z+ab_1+bd_1}{(ca_1+dc_1)z+cb_1+d_1d} \quad \text{— linear fractional transform.}$$

5. (a)

$$\int\limits_c \cos^2 z dz = \frac{2+\sin 2}{4} - i\frac{2+\sinh 2}{4} \approx 0.727 - 1.407i.$$

(b)

$$\int\limits_c z^2 e^{z^3} dz = \frac{e^{-i}-e^8}{3} \approx -993.47 - 0.280i.$$

(c)

$$\int\limits_c \bar{z} dz = 2\pi i \approx 6.28318i.$$

6. (a)

$$\oint\limits_c \left(\frac{z^3}{3z-1} + \frac{1+z}{z-3}\right) dz = \frac{2\pi i}{81} \approx 0.0775i.$$

(b)

$$\oint \frac{(z+2)^4+\cos 2z}{(z+1)^2} dz = 4\pi i(2+\sin 2) \approx 36.559i.$$

(c)

$$\oint\limits_c \frac{\ln(z-2)}{(2z-1)^3} dz = \frac{-\pi i}{18}.$$