MATH 6172 **Test II** **Spring 2002**

Name : _____

SHOW THE DETAILS OF YOUR WORK ID : _____

1. (15%) Find the radius of convergence of the following power series:

   (a) $\sum_{n=1}^{\infty} \frac{2^{10n}}{n!} z^n$;

   (b) $\sum_{n=1}^{\infty} \frac{n(n+1)}{6^n} z^{3n}$;

   (c) $\sum_{n=1}^{\infty} \frac{n^n}{n!} (z+1)^n$.

2. (15%) Expand $f(z) = \dfrac{z}{(z+1)(z-2)}$ in a Laurant series valid for the indicated annular domain:

   (a) $0 < |z+1| < 3$,

   (b) $3 < |z+1|$,

   (c) $1 < |z| < 2$.

3. (10%) Evaluate the following integral (counterclockwise)

$$\oint_C \frac{z^2 \sin z}{4z^2 - 1} dz; \quad C : |z| = 2$$

4. (10%) Evaluate the improper integral

$$\int_{-\infty}^{\infty} \frac{\cos 2x}{(x^2 + 1)^2} dx$$

5. (10%) Evaluate the following integral

$$\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}.$$

6. (10%) Show that $F(z) = z^2$ describes a flow around a corner. Find and sketch the streamlines and find V.

7. (10%) Find the potential $\Phi(r, \theta)$ in the unit disk $r < 1$, having the given boundary values $\Phi(1, \theta) = 3 + 2\cos 2\theta + \cos^2 2\theta$.

8. (10%) Find the electrostatic potential between the two coaxial cylinders $C_1 : |z| = 1$ (potential $U_1 = 0$ volts) and $C_2 : |z| = 5$ ($U_2 = 110$ volts).

9. (10%) Find the temperature field T in the first quadrant of the z-plane if the temperature at the x-axis and the y-axis is given:

$$T = \begin{cases} 10°C, & \text{if } 0 \leq x < 1 \text{ and } y = 0, \\ 0, & \text{if } 1 < x \text{ and } y = 0 \end{cases}$$

and

$$T = \begin{cases} 10°C, & \text{if } 0 \leq y < 1 \text{ and } x = 0, \\ 0, & \text{if } 1 < y \text{ and } x = 0. \end{cases}$$