Math 6172 Answers to Problems in Test II Spring 2002

1. (a)

$$R = \infty.$$

(b)

$$R = 6^{\frac{1}{3}}.$$

(c)

$$R = \frac{1}{e}.$$

2. (a)

$$\frac{z}{(z+1)(z-2)} = \frac{\frac{1}{3}}{z+1} - \frac{2}{9}\sum_{n=0}^{\infty}\left(\frac{z+1}{3}\right)^n.$$

(b)

$$\frac{z}{(z+1)(z-2)} = \frac{1}{(z+1)} + \frac{2}{9}\sum_{n=0}^{\infty}\left(\frac{3}{z+1}\right)^{n+2}.$$

(c)

$$\frac{z}{(z+1)(z-2)} = \frac{1}{3z}\sum_{n=0}^{\infty}\left(\frac{-1}{z}\right)^n - \frac{1}{3}\sum_{n=0}^{\infty}\left(\frac{z}{2}\right)^n.$$

3.

$$\oint_c \frac{z^2 \sin z}{4z^2-1} dz = i\frac{\pi}{4}\sin\frac{1}{2} \approx 0.3765i.$$

4.

$$\int_{-\infty}^{\infty} \frac{\cos 2x}{(x^2+1)^2} dx = \frac{3\pi}{2}e^{-2} \approx 0.63.$$

5.

$$\int_0^{2\pi} \frac{d\theta}{2+\cos\theta} = \frac{2\pi}{\sqrt{3}} \approx 3.63.$$

6. The relation $\psi = 2xy = constant$ gives the streamlines and for $\mathbf{V}$ we have

$$\mathbf{V} = 2x - 2yi.$$

7.

$$\phi(r,0) = \frac{7}{2} + 2r^2 \cos 2\theta + \frac{1}{2} r^4 \cos 4\theta.$$

8.

$$\phi(r) = \frac{110}{\ln 5} \ln r.$$

9.

$$T(x,y) = \frac{10}{\pi} \arg \frac{z^2 - 1}{z^2 + 1}.$$