**MATH 6172** **Test II** **Spring 2003**

Name : ____________________

**SHOW THE DETAILS OF YOUR WORK** ID : ____________________

1. (a) Is the series $\sum_{n=1}^{\infty} \frac{e^{in}}{n^2+2}$ convergent or divergent? (Please give explanations for your answer.)

   (b) Is the series $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$ convergent or divergent? (Please give explanations for your answer.)

   (c) Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{1}{(1+i)^n}(z-i)^n$;

   (d) Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^n}(z+1)^n$.

2. Find the Maclaurin series of the following functions and determine the precise regions of convergence:

   (a) $1/(1+z^2)$.

   (b) $\sin^2 z$.

   (c) $1/(z+i)^2$.

   (d) $\int_0^Z \cos t^2 dt$.

3. Find all the Taylor and Laurent series of the function $f(z) = \dfrac{1}{z+1}$ (assume the center is 1).

4. Evaluate the following integral (counterclockwise)

$$\oint_C \frac{e^z + z}{z^3 - z} dz; \quad C : |z| = 1/2$$

5. Evaluate the following integral

$$\int_0^{2\pi} \frac{1}{2 - \cos\theta} d\theta.$$

6. Evaluate the improper integral

$$\int_{-\infty}^{\infty} \frac{\sin 2x}{x^2 + x + 1} dx$$

7. Find the potential $\Phi(r, \theta)$ in the unit disk $r < 1$, having the given boundary values $\Phi(1, \theta) = 1 + 2\cos 2\theta + 3\cos^2 2\theta$.

8. Find the electrostatic potential between the two portions of a circular plate $C_1 : |z| = 1$ and $\theta \in (0, \pi/2)$ and (potential $U_1 = 0$ volts) and $C_2 : |z| = 1$ and $\theta \in (\pi/2, 2\pi)$ ($U_2 = 110$ volts).

9. Find the temperature field $T$ in the first quadrant of the $z$-plane if the temperature at the $x$-axis and the $y$-axis is given:

$$T = \begin{cases} 10°C, & \text{if } 0 \leq x < 1 \text{ and } y = 0, \\ 0, & \text{if } 1 < x \text{ and } y = 0 \end{cases}$$

and

$$T = \begin{cases} 10°C, & \text{if } 0 \leq y < 2 \text{ and } x = 0, \\ 0, & \text{if } 2 < y \text{ and } x = 0. \end{cases}$$